Feedback Effects on Amplifier Response

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The open-loop gain of an amplifier typically varies in response to parameters that are not very controllable. The use of negative feedback stabilizes the closed-loop gain of the amplifier at a lower and predictable gain level. Negative feedback also widens the frequency response. This article describes the math behind these effects and provides a graphical example. The example shown is for first order amplifier response. All amplifiers will ultimately show higher order effects and the associated accumulative phase shifts can result in negative feedback becoming positive feedback at some frequencies. It is possible to apply enough negative feedback to many amplifiers such that they oscillate.

The amplifier model used has an open-loop gain, Av, at the mid-band frequency and finite low cutoff (associated with time constant T_1) and high cutoff (associated with time constant T_2) frequencies as represented by the resistor-capacitor networks. For simplicity in the math the two RC sections are isolated from each other and from source and load impedances so that there is no interaction – only the direct time constants affect the amplifier frequency response. Note that these RC sections are a first-order model of the amplifier's frequency response effects and that in a real amplifier these RC sections may not explicitly exist. The model for this amplifier is shown in Figure 1.

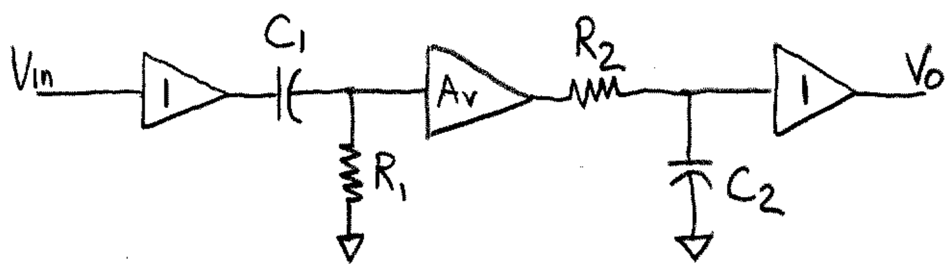

Figure 1: Generic amplifier with first order effects

The low cutoff frequency is determined by R_1 and C_1. The high-pass transfer function of that network is:

$$L_{(s)} = \frac{R_1}{1/C_1 s + R_1} = \frac{R_1 C_1 s}{R_1 C_1 s + 1} = \frac{T_1 s}{T_1 s + 1} \qquad \text{Eq. 1}$$

The high cutoff frequency is determined by R_2 and C_2. The low-pass transfer function of that network is:

$$H_{(s)} = \frac{1/C_2 s}{R_2 + 1/C_2 s} = \frac{1}{R_2 C_2 s + 1} = \frac{1}{T_2 s + 1} \qquad \text{Eq. 2}$$

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The open loop transfer function of the amplifier is by combining the above transfer functions and including the open-loop gain.

$$G_{(s)} = \frac{A_v T_1 s}{(T_1 s + 1)(T_2 s + 1)} \qquad \text{Eq. 3}$$

The closed-loop transfer function of the amplifier with feedback factor, B (varies between 0 and 1) is found by using Mason's gain formula applied to the circuit in Figure 2.

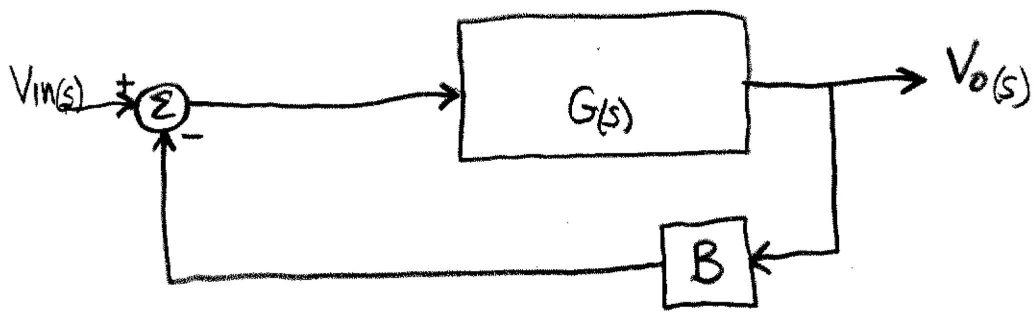

Figure 2: Block diagram model of closed-loop amplifier

$$Q_{(s)} = \frac{\dfrac{A_v T_1 s}{(T_1 s + 1)(T_2 s + 1)}}{1 + \dfrac{B A_v T_1 s}{(T_1 s + 1)(T_2 s + 1)}} = \frac{A_v T_1 s}{(T_1 s + 1)(T_2 s + 1) + B A_v T_1 s}$$

$$= \frac{A_v T_1 s}{T_1 T_2 s^2 + (T_1 + T_2 + B A_v T_1)s + 1} = \frac{A_v s / T_2}{s^2 + (1/T_1 + 1/T_2 + B A_v / T_2)s + 1/(T_1 T_2)} \qquad \text{Eq. 4}$$

The mid-band frequency is when $w = \mathrm{sqrt}(1/(T_1 T_2)$. Eq. 5

The magnitude, M, of the mid-band response is found by substituting the mid-band frequency from Eq. 5 into Eq. 4.

$$M = \frac{A_v / T_2}{(1/T_1 + 1/T_2 + B A_v / T_2)} = \frac{1}{[(T_2/T_1) + 1]/A_v + B} \qquad \text{Eq. 6}$$

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If the open-loop gain is high compared to 1/B and the high cutoff frequency is high compared to the low cutoff frequency (i.e. $T_2 << T_1$) then the mid band gain asymptotically approaches 1/B. This illustrates the advantage of negative feedback in stabilizing the closed-loop gain of an amplifier to a fixed and readily predictable value.

The other effect of negative feedback is to extend the lower and upper cutoff frequencies. The exact cutoff frequencies can be found by equating Eq. 4 to the mid-band response reduced by 3 dB – the sqrt(2) factor.

$$\left| \frac{A_v s/T_2}{s^2 + (1/T_1 + 1/T_2 + BA_v/T_2)s + 1/(T_1T_2)} \right| = \frac{1}{\{[(T_2/T_1) + 1]/A_v + B\}*\mathrm{sqrt}(2)} \qquad \text{Eq. 7}$$

jw is substituted for s and for magnitude purposes the j in the numerator is ignored and the denominator is evaluated as the square root of the sum of the squares of the real and imaginary parts. Both sides are squared to eliminate the square root and a quadratic equation in w^2 is established – there will be w^4, w^2, and w^0 terms. Use standard methods to solve for w^2 then take the square root. One of the roots will be the low cutoff frequency and the other root will be the high cutoff frequency.

The details of this algebra are left as an exercise for the reader. Once the reader understands the algebra, the simple approximate conclusion can be reached that the bandwidth increases in proportion to the mid-band gain reduction. If the amount of negative feedback reduces the mid-band gain by half then the new low cutoff frequency is about half of the original and the new high cutoff frequency is about twice the original.

This is illustrated in Figure 3 which models an amplifier with an open loop gain of 1,000 and which has a low cutoff frequency of 200 Hz and a high cutoff frequency of 5,000 Hz. The frequency response based on Eq. 4 is plotted for a variety of feedback factors, B, from 0 (open-loop) to 0.1 (a gain of 10). Observe that the highest curve is the open loop response and the 200 and 5,000 Hz cutoff frequencies are easily seen. Observe that as the feedback factor is increased the mid-band gain is reduced and the frequency response widens. Observe that when the mid-band gain is reduced by a factor of 100 to 10 the low cutoff frequency becomes 2 Hz and the high cutoff frequency becomes 500,000 Hz, also factors of 100 from the original.

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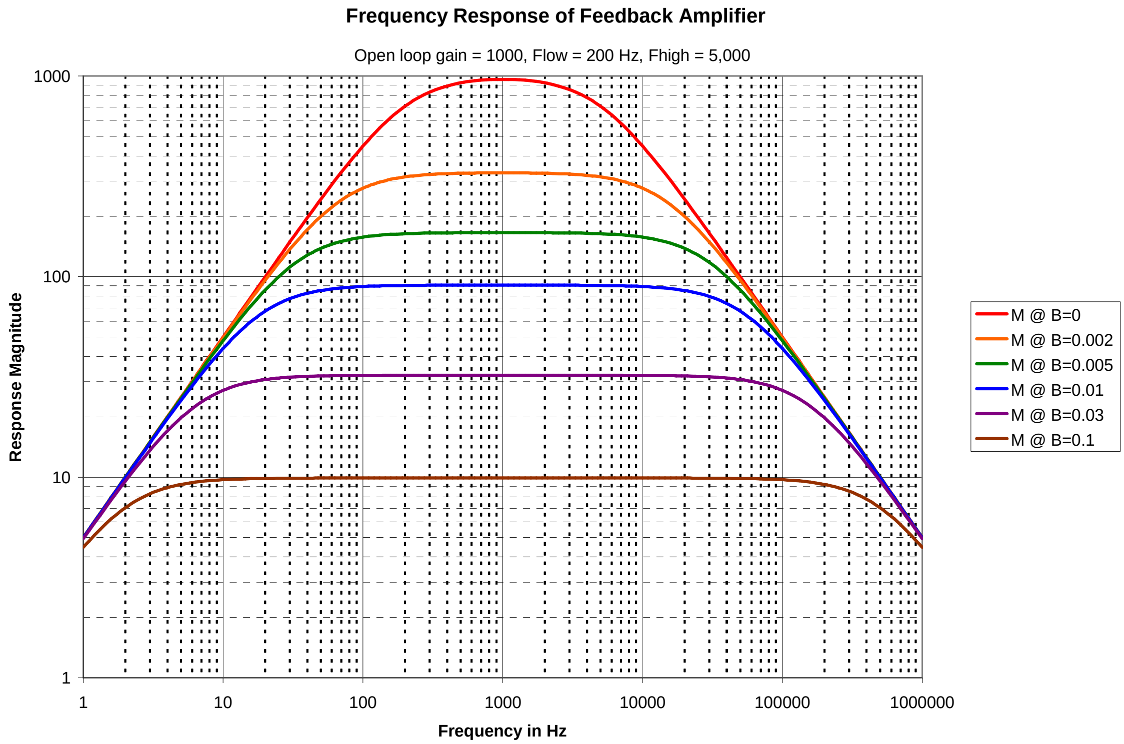

Figure 3: Frequency response of example amplifier