The Hall Network
A one-pot Tunable Notch Filter

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This note has been superseded by the following article I wrote for Electronic Design. It was published Jan. 31, 2012 on the online version of Electronic Design. That article has much more treatment.

http://electronicdesign.com/article/analog-and-mixed-signal/Rediscover-The-Truly-Tunable-Hall-Network-.aspx

I will keep the following here for the time being but really there is no point. Do note that the optimum choice for capacitors is all capacitors the same value. Disregard the hand note below about C1 = C3 = 10C2.

There is a passive RC circuit known as the Hall Network that is similar to the twin or parallel tee circuits except that it has the interesting and useful properties of a single potentiometer controls the tuning and the input and output signals are also ground based. This circuit was presented by Henry P. Hall in the September, 1955 of the IRE Transactions on Circuit Theory on pages 283 and 284 (see Figure 2). I accidently stumbled across this circuit in the mid 1970s and was intrigued by its properties.

The circuit is shown in Figure 1 along with basic design equations. Many other cases are possible.

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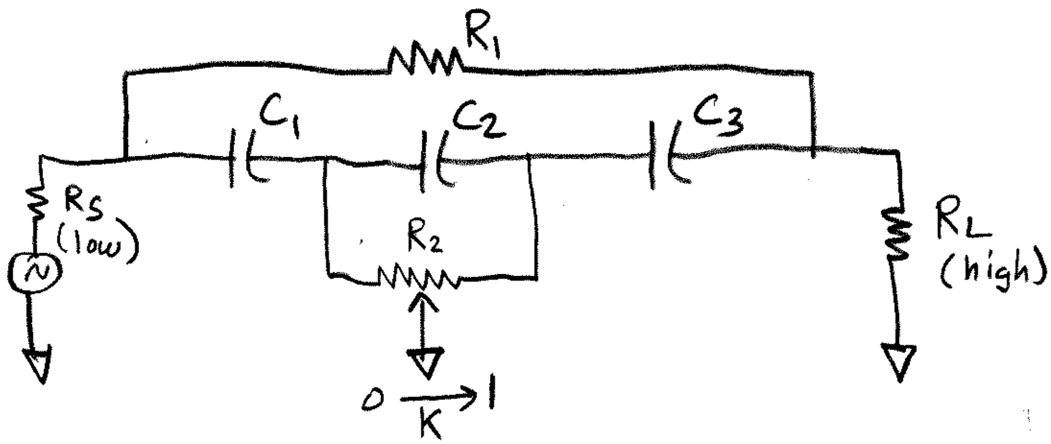

Let $C_1 = C_3 = 10\,C_2$

$R_1 = 13.2\,R_2$

$$w_{notch} = \frac{1}{2R_2C_2\sqrt{30K(1-K)}}$$

$$w_{notch_{min}} = \frac{1}{5.48\,R_2C_2} \qquad (K=0.5)$$

Figure 1: Hall Network

The tuning is very non-linear and the minimum notch frequency is when the pot is centered. The frequency of the notch theoretically increases to infinity as the pot approaches either end away from center although that is not practical. In practical application some minimum resistance might be used on either end of the pot to limit the extreme tuning or a resistance equal to that of the pot could be used at one end so that the full range of the pot can be utilized.

The circuit is very easy to use although being a third order network with a bridged node makes symbolic analysis very tortuous. A scanned copy of the arduous analysis I performed in 1979 is in a related portion of the web site of this document. Symbolic analysis is necessary in order to work out and optimize the design equations.

I have a lot more information planned but right now I am time limited. I will show frequency response plots and other design details as I have time to add to this document. Using amplifiers, the circuit can be converted into a tunable band-pass filter although control of Q is limited.

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Tuning

As shown in Figure _ the tuning range for the circuit shown in Figure 1 is from a theoretical infinity when the potentiometer is at the extreme counter-clockwise position to a minimum frequency when the potentiometer is at the center of rotation and then increasing back towards a theoretical infinity when the potentiometer is at the extreme clockwise position. The tuning is very non-linear and the practical tuning limit is a frequency range of about 4 to 1. Initially it seems obvious to place a series resistor equal to the resistance of the potentiometer as shown in Figure _ so that the extreme counter-clockwise position of the potentiometer represents the minimum frequency. It also seems obvious to place another series resistance at the clockwise end of the potentiometer to limit the tuning range to something useful. These are both good things to do but raise the question of whether these parts can be chosen such as to make the tuning more responsive at the lower frequency end.

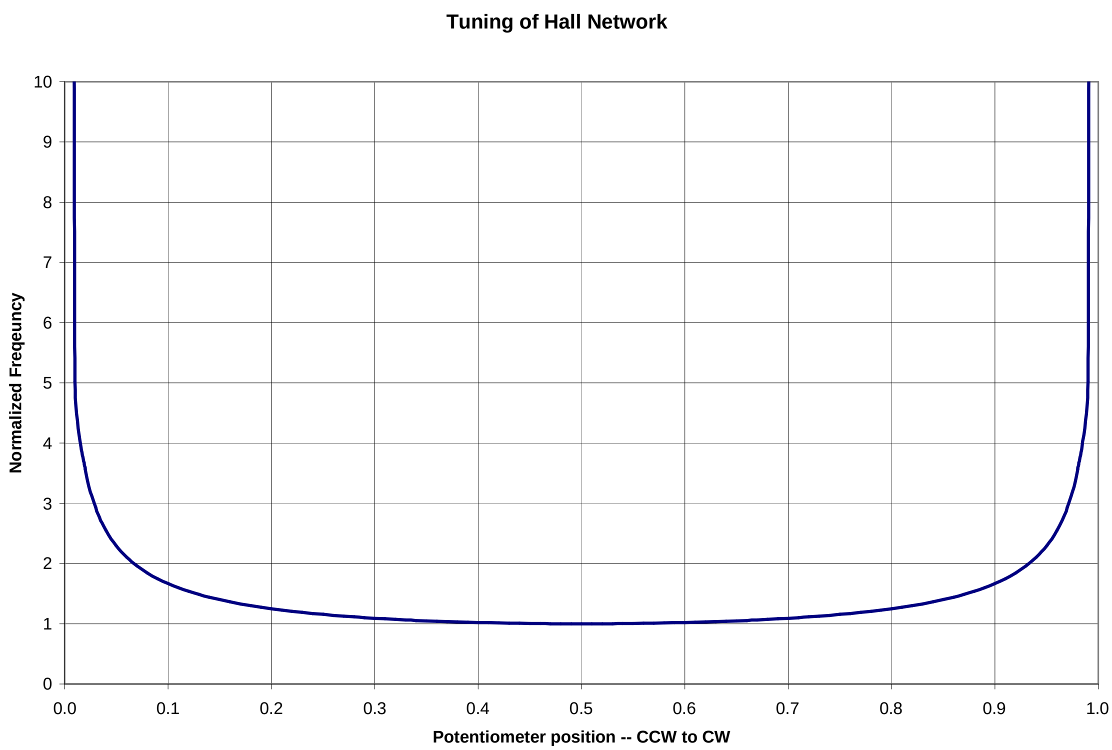

Figure _: Tuning of Hall Network

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Sept – 1955

IRE TRANSACTIONS—CIRCUIT THEORY

RC Networks with Single-Component Frequency Control*

The subject of single-component control of oscillators and selective amplifiers, as described in Mr. Clothier's article in the March, 1955, issue of the *PGCT Transactions*, has been of great interest to us. We have come across several networks of this type which, to my knowledge, have not appeared in the literature.

The first network (Fig. 1) is somewhat

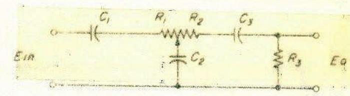

Fig. 1

similar to a Wien bridge, but requires one more condenser. For a null, this network would form two arms of a bridge, the other two being a fixed voltage divider. For this network, if $C_1 = C_3$.

$$\frac{E_o}{E_{in}} = \frac{1}{1 + \frac{C_2}{C_1} + \frac{(R_1+R_2)}{R_3}\left(1 + \frac{C_2}{C_1}\right) + j\left\{\omega R_1 C_2\left(1 + \frac{R_2}{R_3}\right) - \frac{1}{\omega C_1 R_3}\left(2 + \frac{C_2}{C_1}\right)\right\}}$$

Notice that the magnitude at peak is independent of the value of R_1 or R_2 as long as $R_1 + R_2$ is constant.

The frequency of the peak is given by

$$\omega^2 = \left(2 + \frac{C_2}{C_1}\right)\frac{1}{C_1C_2R_1(R_2+R_3)}.$$

By taking the topographical dual and interchanging R's and C's and also input and output, we arrive at the second network (Fig. 2) which uses a differential capacitor.

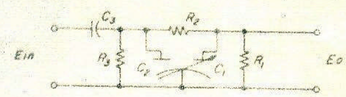

Fig. 2

The expressions for E_o/E_{in} and frequency can be obtained from the above expressions by interchanging R's and C's, but keeping the subscripts the same. (Here R_1 has to equal R_2, and $C_1 + C_2$ must be constant in order to keep the magnitude at peak constant.)

The next network gives a complete null,

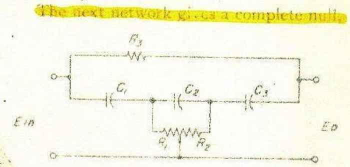

Fig. 3

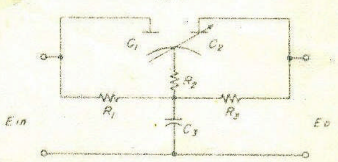

Fig. 4

* Received by the PGCT, April 25, 1955.

and yet has a terminal common to input and output. Here if $C_1 = C_3$, the null conditions can be written

$$R_1R_2\omega^2 = \frac{(C_1+C_2)(R_1+R_2)}{C_1^2C_2R_3}$$

$$= \frac{1}{C_1(2C_2+C_1)}$$

Since $R_1 + R_2$ is constant, the null conditions are met as R_1/R_2 is varied while the null frequency is changed.

The last network (Fig. 4) is the "dual" of Fig. 3, and the null conditions are given by the above expression if the R's and C's are interchanged.

Henry P. Hall
General Radio Company
Cambridge, Mass.

Figure 2: Copy of IRE Transactions showing Hall Network (figure 3 in article) shown via the "Fair use" doctrine of Copyright law

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My hand derivation of the circuit is shown in hall_network.jpg which is on my web site: http://www.kennethkuhn.com/electronics . Those notes are grouped via a sequence of seven numbered parts. A description of those parts is shown below.

1. This section shows the fundamental circuit with the sum of R1 and R2 equal to the pot. A pi to tee conversion is performed on the circuit excluding R3 in order to significantly reduce the number of components. This type of network conversion is about the only way to attach bridged-node circuits.

2. R3 is still part of a bridged-node. The tee from (1) is converted back to a pi (but a different pi than previous) to eliminate the bridge so that R3 can be included. The end result of this leads to developing the transfer function in step 3.

3. The transfer function is developed using voltage divider theory on the previous result. This third order transfer function is then solved for the notch frequency – the circled equation. The imaginary terms (third and first order) must be zero at the notch frequency. The real terms (second and zero order) also must be zero at the notch frequency. The solution to both equations results in two w^2 terms that must be equal. The solution to that relates the component values to the notch frequency. A popular simplification is substituted: Let C1 = C3 and C1 = 10*C2. This leads to R3 being equal to 13.2*(R1+R2). Many other simplifications are possible as developed in the next step.

4. The simplifications in (3) are replaced with a generic equation to relate ratios.

5. These are the analytical equations which can be inverted for design.

6. This is the special case transfer function for the simplifications in (3).

7. This is the results of (6) further simplified to relate the notch frequency directly to the pot position, k which can vary from 0 to 1 over the extremes of position.