Managed Alias Sampling

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Introduction

This article discusses a method to manage rather than fight a sampling alias. It is customary to include a low-pass filter to reduce out of band signals that will alias into the baseband via periodic sampling. In very low frequency analog systems (i.e. the frequency spectrum of interest is from DC to maybe ten Hertz or so) the dominant out of band signal is usually the power line fundamental frequency, either 50 or 60 Hz. The sampling frequency is typically from a few Hz to perhaps twenty Hz with ten Hz being a very popular choice. Without a low-pass filter it is clear that the power line frequency will show up as a low frequency alias. Using standard methods one could design a low-pass filter that could reduce that alias to an acceptable level. However, in multichannel applications that becomes a lot of filters. An alternate method might be to over sample all the channels and implement a digital low-pass filter in a computer. That works but requires a lot of computation. One almost clever method is to choose a sample frequency such that a notch filter is naturally formed at the power line frequency thus alleviating the need for a low-pass filter. This article shows that that method fails (the alias is a significant amplitude sine wave near DC that is impossible to remove) but that a modification to the method succeeds using only a simple low-pass/notch FIR filter in the computer.

I have worked with low frequency systems for years and have observed many alias problems involving power line pickup. One of the earliest problems noted some decades ago was using a hand-held Fluke multimeter that had a conductance function for measuring high resistance values. It was often seen that if one watched the meter for several minutes that the reading would periodically go up and down over the course of half a minute or so. Those type meters use a dual slope integrating type of analog to digital converter where the integration period is cleverly chosen to be an integer number of cycles of the power line frequency thus forming a notch filter that in theory completely eliminates power line interference. The problem is that the power line frequency is not a precise value and can vary plus or minus a small fraction of a percent (I have never found actual specifications but various things I have read give me the impression that for 60 Hz in the United States that the vast majority of the time the short term error is no more than about +-0.05 Hz). Thus, a fractional Hz signal is produced that explains the periodic wander. The non perfect power line frequency is on the steep slope of the notch filter and the amplitude is significantly reduced so the concept has merit.

I have seen a number of systems that used a sample frequency of ten Hz. That frequency is very convenient but is one of the worst possible choices since like in the multimeter example the alias is very near DC. When such sampled data is plotted it is common to see a sine wave with a period of tens of seconds. The result is often a wild goose chase for non-existent thermal issues as what else could have such a low frequency. Figure 1 shows a typical result. The signal is a simulated series of three peaks representative of a detector on a gas chromatograph. The sampling frequency is 10 Hz and the power line frequency is 60.02 Hz which creates an alias with a 50 second period. The amplitude of the power line pick up is 1 volt peak. Note the low frequency on the baseline of the signal. This could be confused with other issues instead of aliasing. I should point out that even if the power line frequency were exactly 60 Hz that an undesirable DC alias (i.e. offset) would occur depending on the relative phase.

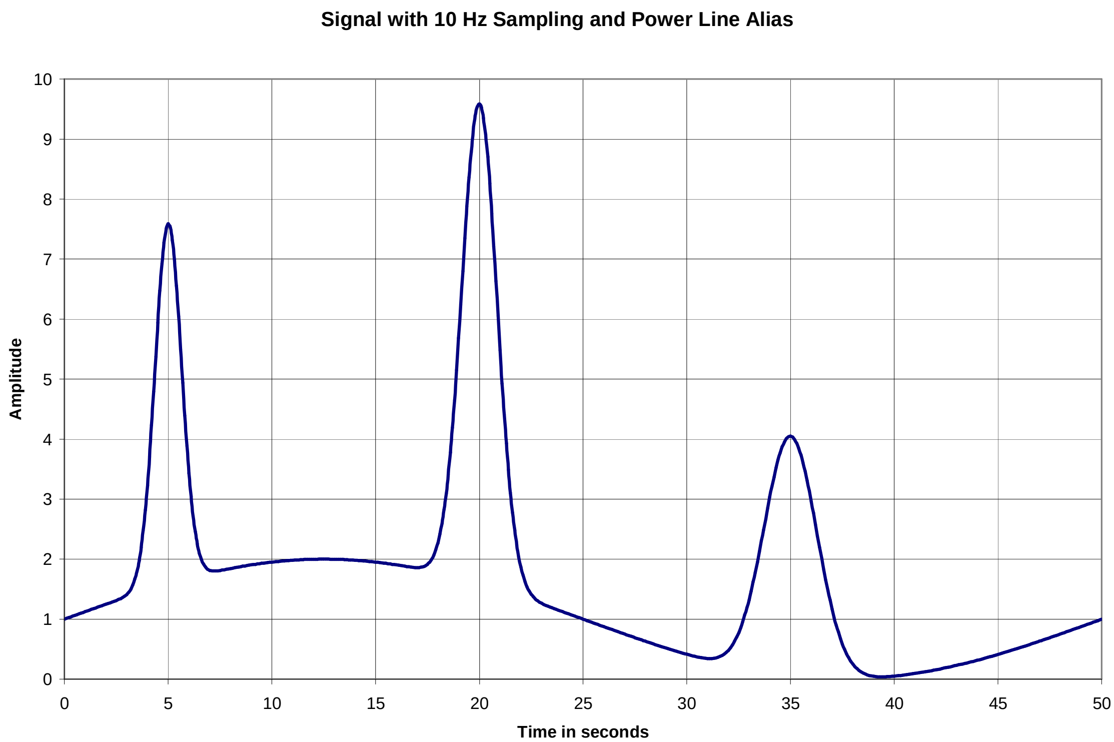

Figure 1: Wandering Baseline Caused by Low Frequency Power Line Alias

The solution

If an alias is going to occur then the ideal place is near the upper frequency of the baseband frequency spectrum where it can be easily removed with a digital notch filter. The worst place for an alias to occur is near DC where no method can remove it (in every instance I have worked on, DC was part of the data so a high-pass filter can not be used). A complication is that the power line frequency may be either 50 or 60 Hz. Intuitively, a good sample frequency for one is not likely to be a good sample frequency for the other. Some equipment has a 50/60 Hz switch to optimize the line frequency rejection. However, a little study shows that there is a set of “magic” frequencies that work identically for either line frequency.

Figure 2 shows a plot of the alias frequencies for both 50 and 60 Hz line frequencies. Each set of lines was plotted using Equation 1 which selects the minimum of two possible alias outcomes.

$$F_A = MIN(F_P - INT(F_P/F_S)*F_S, (INT(F_P/F_S) + 1)* F_S - F_P) \qquad \text{Eq. 1}$$

where
F_A = alias frequency in Hz
F_P = power line frequency in Hz
F_S = sample frequency in Hz

A poor choice of sample frequency is when the alias frequency is at or near DC. A good choice of sample frequency is when the alias frequency is high – approaching $F_S/2$.

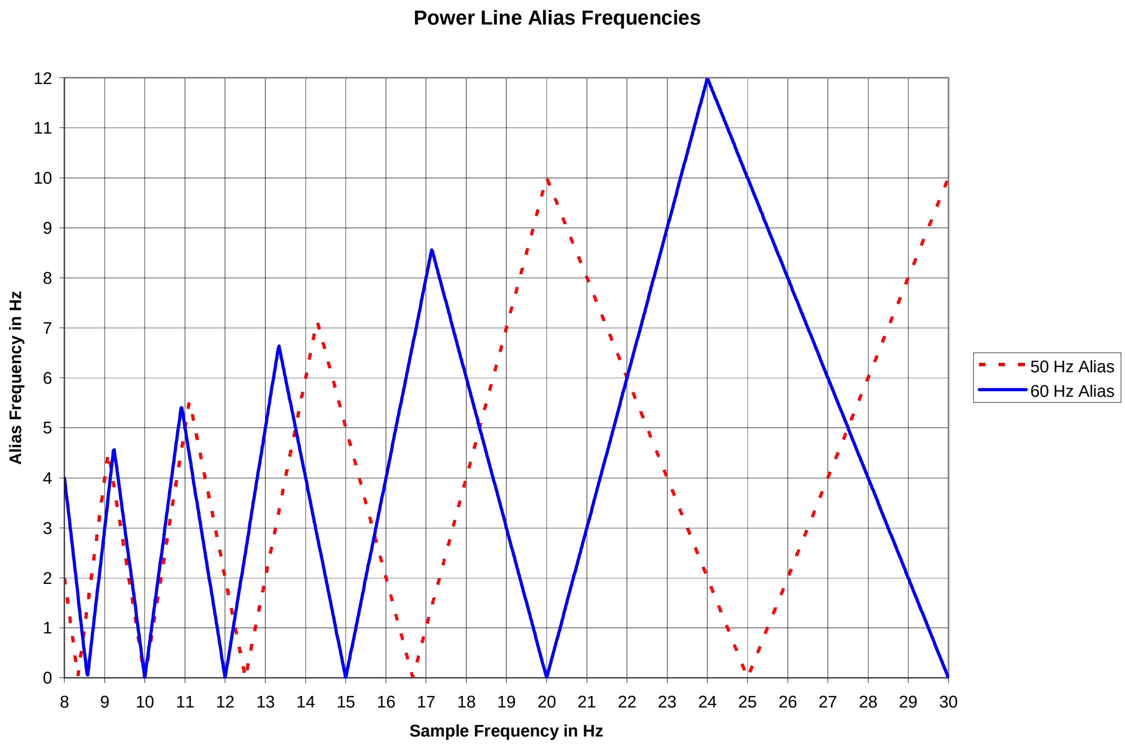

Figure 2: Power Line Alias Frequencies

It can be observed that some sample frequencies result in the identical alias frequency whether the power line frequency is 50 or 60 Hz. These are the "magic" frequencies. The best ones to use are where the alias frequency is high. These frequencies can be found by solving for the equal condition of the two terms in Equation 1 for the two power line frequencies. The solution set for all matching alias frequencies over the range plotted is given in Table 1.

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Sample Frequency	Alias Frequency	Comments
8.461538 Hz	0.769 Hz	Poor
9.166667	4.167	Good
10.000000	0.000	Bad
11.000000	5.000	Good
12.222222	1.111	Poor
13.750000	5.000	Good
15.714286	2.857	Poor
18.333333	5.000	Good
22.000000	6.000	Excellent, my favorite
27.500000	5.000	Good

Table 1: Sample and Alias Frequencies for either 50 or 60 Hz Power Line

The 22 Hz sample frequency has the nice characteristic of a high alias frequency (6 Hz) and that if an eleven stage FIR averaging filter is employed, there is a notch in the frequency response at 6 Hz. So elimination of the alias is easy. Some clever techniques on some of the other "good" frequencies can also achieve good results but that is an exercise for the reader.

Figure 3 shows the result of 22 Hz sampling on the original data with a 1 volt peak 60 Hz line frequency interference.

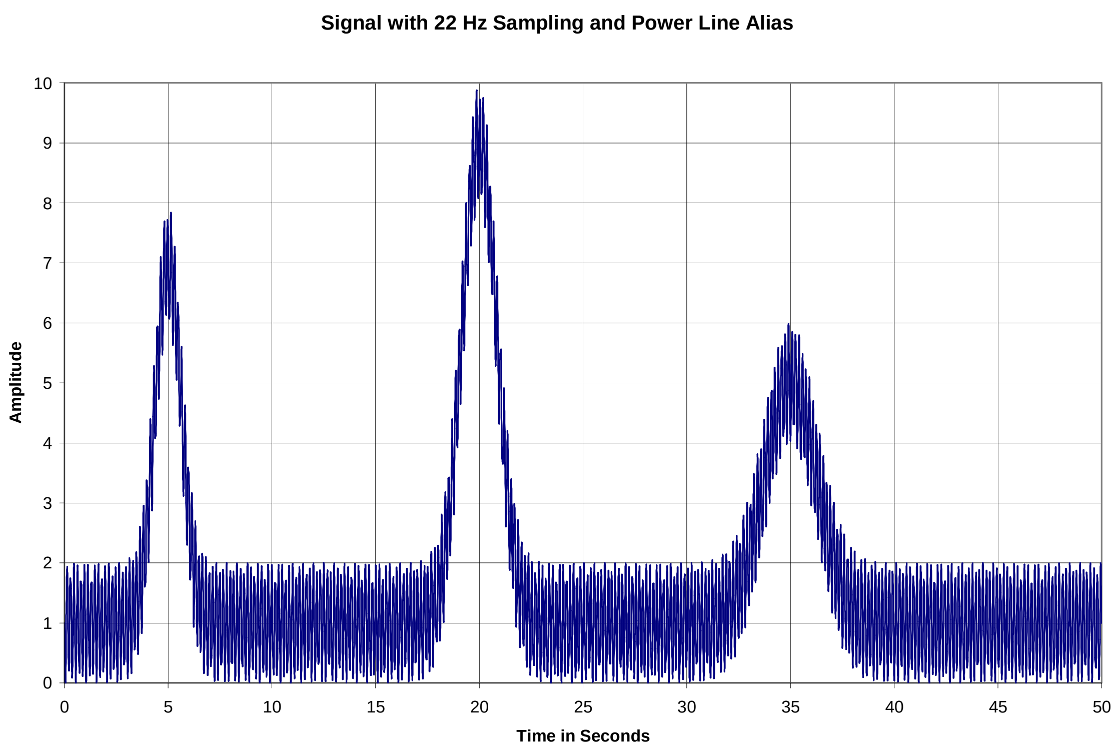

Figure 3: Signal with 6 Hz Power Line Alias

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Figure 4 shows the result of an eleven stage FIR averager that notches out the 6 Hz alias. The desired signal is barely affected but the alias is gone.

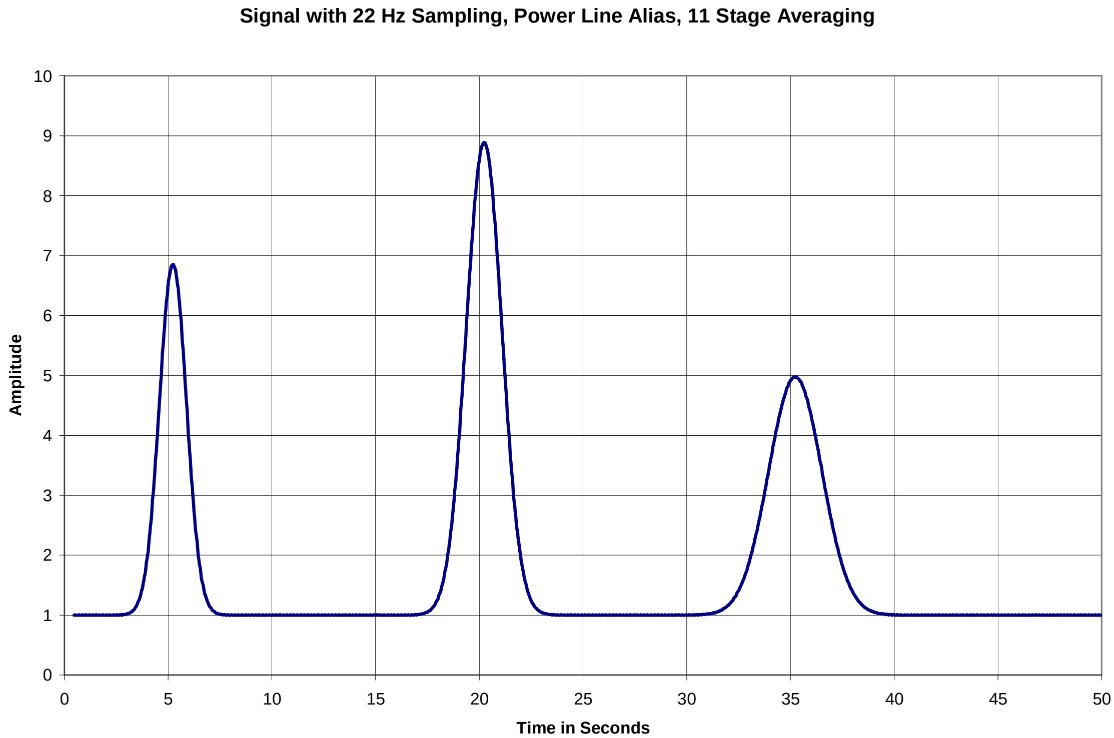

Figure 4: Data from Figure 3 after Eleven stage FIR Averager

In reality the power line frequency will be close to but not exactly 60 Hz. We need to know how effective this process is for those cases. As an extreme example, suppose the power line frequency was 60.5 Hz which is an error beyond anything that should ever be seen. This would shift the alias frequency down to 5.5 Hz which misses the notch although it is still on the steep slope. However, the filter is still effective as seen in Figure 5. The power line frequency is rarely off more than about 0.02 Hz and a plot of that is indistinguishable from Figure 4.

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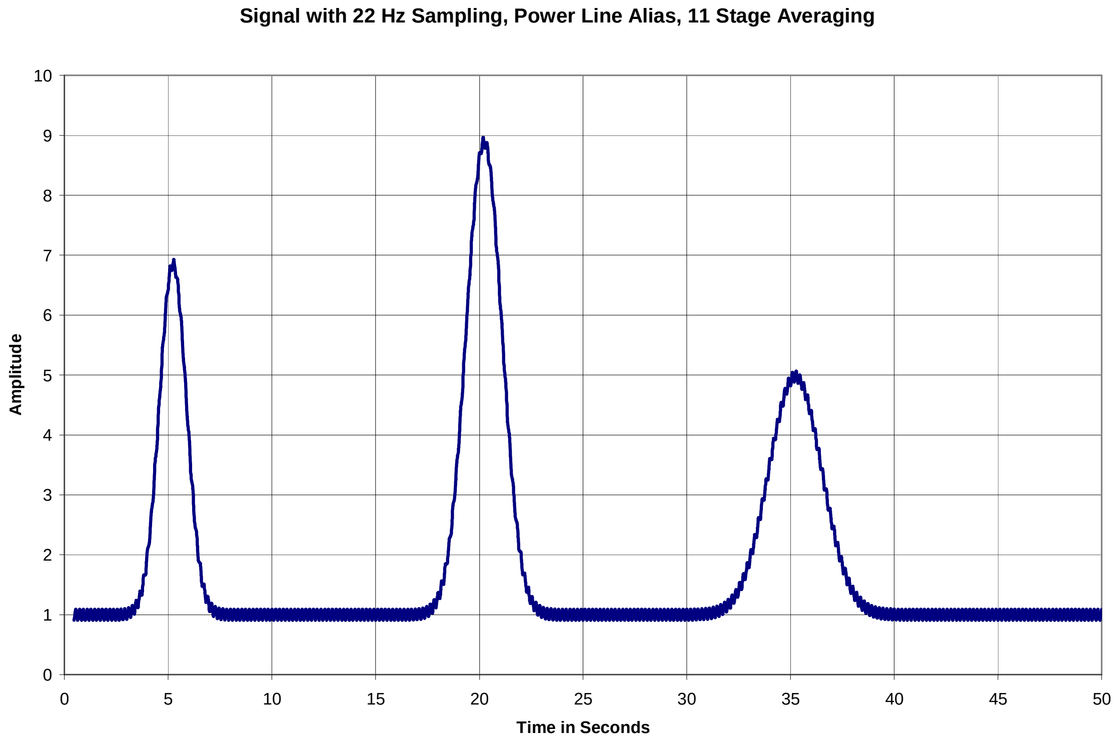

Figure 5: 22 Hz Sampling with 60.5 Hz Power Line and Eleven Stage FIR Averager

Conclusion

Rather than being arbitrary, the sample frequency should be thoughtfully chosen so that known aliases occur in a relatively harmless portion of the baseband spectrum.