2 ELEMENT COMPLEX IMPEDANCE MATCHING CIRCUIT

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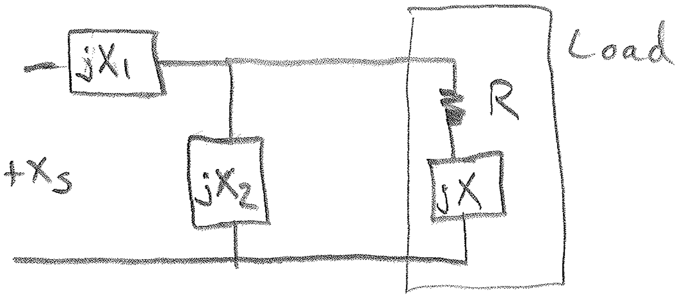
3-9-07



$Z_S = R_S + X_S$

$$jX_2 \,\|\, (R + jX) = \frac{jX_2R - X_2X}{jX_2 + R + jX} = \frac{-X_2X + jX_2R}{R + j(X_2 + X)}$$

$$\frac{-X_2X + jX_2R}{R + j(X_2+X)} \times \frac{R - j(X_2+X)}{R - j(X_2+X)} = \frac{-X_2XR + jX_2R^2 + jX_2X(X_2+X) + X_2R(X_2+X)}{R^2 + (X_2+X)^2}$$

$$= \frac{X_2R(X_2 - X) - X_2XR + j\left(X_2R^2 + X_2X(X_2+X)\right)}{R^2 + (X_2+X)^2}$$

$$= \frac{X_2^2R}{R^2 + (X_2+X)^2} + \frac{jX_2\left(R^2 + X(X_2+X)\right)}{R^2 + (X_2+X)^2}$$

Set $R_S = \dfrac{X_2^2 R}{R^2 + (X_2+X)^2}$ AND SOLVE FOR X_2

$R_SR^2 + R_S(X_2+X)^2 = X_2^2R$

$R_SR^2 + R_SX_2^2 + 2R_SXX_2 + R_SX^2 - X_2^2R = 0$

$(R_S - R)X_2^2 + (2R_SX)X_2 + R_S(R^2 + X^2) = 0$

Use quadratic formula to solve for X_2. Both results are valid

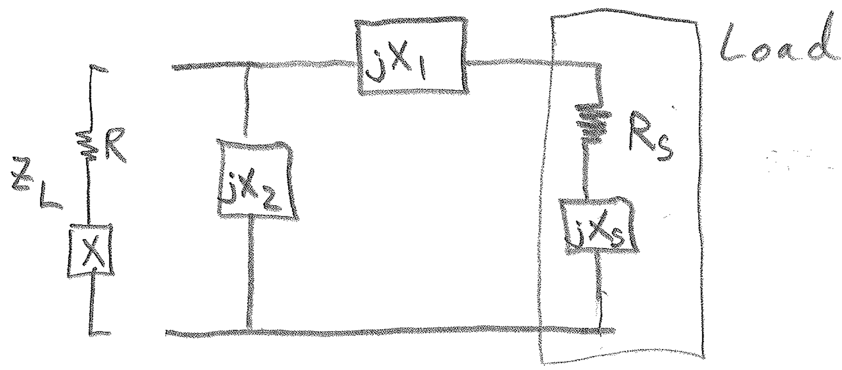
Use X_2 to solve for X_1

$$X_S = \frac{X_2\left(R^2 + X(X_2+X)\right)}{R^2 + (X_2-X)^2} + X_1$$

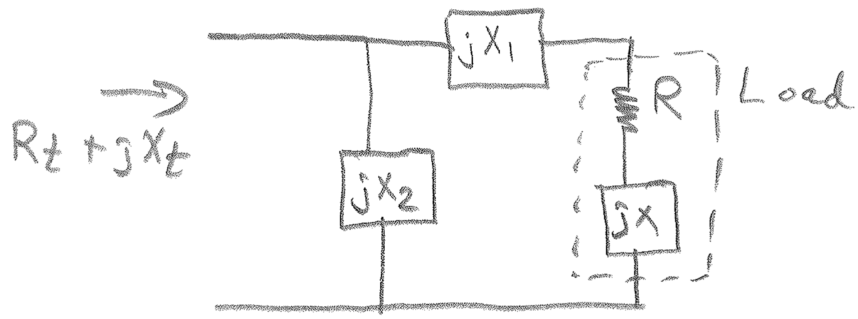
$$\boxed{X_1 = X_S - \frac{X_2\left(R^2 + X(X_2+X)\right)}{R^2 + (X_2-X)^2}}$$

There are two solutions using the two different values of X_2 — use the solution with the most practical values

Another set of two solutions exists by swapping Z_{load} for Z_{source}. The network becomes



Thus, four solutions are possible. Compute all four and use the one with the most practical component values. Depending on conditions, one set of solutions may not work since the square-root of a negative number might occur.



$$R_t + jX_t = jX_2 \,\|\, \left(R + j(X_1 + X)\right)$$

$$= \frac{jX_2 R - X_2(X_1+X)}{R + j(X_1+X_2+X)} \times \frac{R - j(X_1+X_2+X)}{R - j(X_1+X_2+X)}$$

$$= \frac{jX_2R^2 + X_2R(X_1+X_2+X) - X_2R(X_1+X) + jX_2(X_1+X)(X_1+X_2+X)}{R^2 + (X_1+X_2+X)^2}$$

$$\boxed{R_t = \frac{X_2^2 R}{R^2 + (X_1+X_2+X)^2}} \quad , \quad \boxed{jX_t = \frac{jX_2\left(R^2 + (X_1+X)(X_1+X_2+X)\right)}{R^2+(X_1+X_2+X)^2}}$$

For Design, treat $X_1 + X$ as a single entity