Crystal Radio Engineering
Audio Transformer

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Review of transformers

As applied to audio systems a transformer is used to transfer a signal at one impedance to another impedance with minimal loss of power. As applied to crystal radios the purpose of the audio transformer is to transform the low impedance of the headphones (typically in the 8 to 30 ohm range) up to the high impedance (typically in the 10,000 to 50,000 ohm range) required for the load on the diode detector.

As a quick review of transformers, the turns ratio, N, is the number of turns on the primary, Np, (or input side) to the number of turns on the secondary, Ns, (or output side). Expressed as an equation,

$$N = Np / Ns \qquad \text{Eq. 1}$$

The ratio of the voltage applied to the primary to the voltage developed (under no load and assuming no losses) on the secondary is also the turns ratio.

As a simple example, a certain 120 VAC power transformer might have 3000 turns on the primary and 150 turns on the secondary. The turns ratio is 20. The secondary voltage would be 6 volts and the voltage ratio is also 20. A real transformer has losses and would be designed such that the loaded secondary voltage is 6. The unloaded voltage is typically ten to twenty percent higher in small transformers.

The current ratio is the reciprocal of the voltage or turns ratio. Thus, the secondary current is N times the primary current for a lossless transformer.

An important transformer parameter for crystal radios is the impedance ratio which by definition is the impedance on the primary divided by the impedance on the secondary. Using the above relations for a lossless transformer we can write

$$\frac{Vp}{Ip} = \frac{Vs * N}{Is / N} \qquad \text{Eq. 2}$$

which simplifies to

$$Zp = Zs * N^2 \qquad \text{Eq. 3}$$

Thus, the impedance ratio is

$$Zp / Zs = N^2 \qquad \text{Eq. 4}$$

Design of transformers

Although the design of a transformer is beyond the scope of this book, it is useful to be aware of the issues so that one understands how to choose and apply a transformer. The design of a transformer consists of determining the required number of turns for the primary for a given size and characteristics of a magnetic core and for a particular low frequency. The turns of wire on the primary forms an inductor whose reactance at the low frequency of interest should be negligibly high but not too high. There is not a single solution so a range of designs are possible with various trade-offs. A transformer with too few turns on the primary will have high core losses and a transformer with too many turns on the primary will have high ohmic losses. There is an optimum number of turns for a particular application. In the case of power transformers the low frequency would either be 50 or 60 Hz and the number of primary turns would be optimized for that.

In the case of an audio transformer we are interested in a range of frequencies – typically from around 100 Hz to 5 kHz for simple systems. The issue for audio transformers is that the optimum number of turns for 100 Hz is too many turns for 5 kHz. The design of an audio transformer involves a complicated trade-off between low-frequency response and high-frequency response. A crude rule-of-thumb is that the inductive reactance of the primary should be around three times the operating primary impedance at the lowest frequency of interest. Then the number of secondary turns is calculated from the desired impedance ratio and then increased a little to compensate for losses.

Can AC power transformers work as audio transformers?

It is hard to find audio transformers with the right impedance ratio for use in crystal radios. It is tempting to look for other types of transformers that might be suitable. One concept is to consider the use of an AC power transformer. Although an AC power transformer is not optimum for audio it has the potential to work to some extent if the turns ratio is in the right range. The turns ratio needs to be in the 30 to 60 range in order to transform a 16 ohm headphone impedance up to several tens of thousands of ohms. This can be achieved with a 120 VAC primary and a 2 – 4 volt secondary. Another possibility is a 240 VAC primary and a 4 – 8 volt secondary. Dual primary/secondary power transformers are common. You might find a transformer with a 120/240 VAC primary with a secondary voltage of 12 if connected in series or 6 if connected in parallel. Use the series connected primary and parallel connected secondary. This provides a nominal turns ratio of 40 which is in the range we are looking for. However, it is important that the transformer design impedance be similar to the impedance we will be working with – this means that the power rating of the transformer should be in a certain range. The typical load impedance for a speaker or headphones is in the 8 to 30 ohm range. Using a 6 volt secondary and a nominal load impedance of 16 ohms then the transformer should have a Volt-Amp rating of around $6^2 / 16 = 2.25$. That is a very small transformer but they are available. This is not a number you have to precisely achieve – it is only a guide to choosing a transformer. Larger transformers in the 5 VA range will also work for this example. You prefer a transformer that will have no unused windings as those will have higher losses – but they are better than nothing so use what you have.

At low audio frequencies an AC power transformer of the right size should work well. At high audio frequencies the transformer will probably not work well because of losses. However, it only has to work good enough for the job – if it enables you to hear a signal then it is working alright.

Cascading transformers

You can cascade two transformers to obtain a higher impedance transformation than you might find in a single transformer. However, the losses are about double that of a single transformer. You could consider more than two transformers but be aware that losses increase with the number of transformers. One approach that can work is shown in Figure 1.

Figure 1: Cascading transformers

Characterizing your transformer

Before you use your transformer you should take some measurements to understand it. Basic characterization data for audio transformers is typically done at a test frequency of 1 kHz. You generally want to know what the nominal primary and secondary impedances are, the power loss, and the frequency response. The following sections describe how to make these measurements.

Measuring the secondary inductance

The first thing to do is to measure the inductance of the secondary winding. We measure the secondary because it is a lower inductance and is easier to measure without various errors creeping in. If we know the inductance of the secondary then we have a good idea of the resistive load impedance that will work best. Use the setup in Figure 2. Use a test frequency of 1 kHz and set the output amplitude of the generator to about 0.6 volts peak-peak as read on scope channel 'A'. Adjust the value of R until the amplitude on scope channel 'B' is between about 40 and 75 percent of the amplitude on scope channel 'A'. This range provides good measurement resolution and accurate results. Record the following measurements on the oscilloscope:

- V_A – the peak-peak applied voltage
- V_B – the peak-peak voltage across the transformer
- θ – the phase angle of scope trace 'B' relative to scope trace 'A' (should be positive)

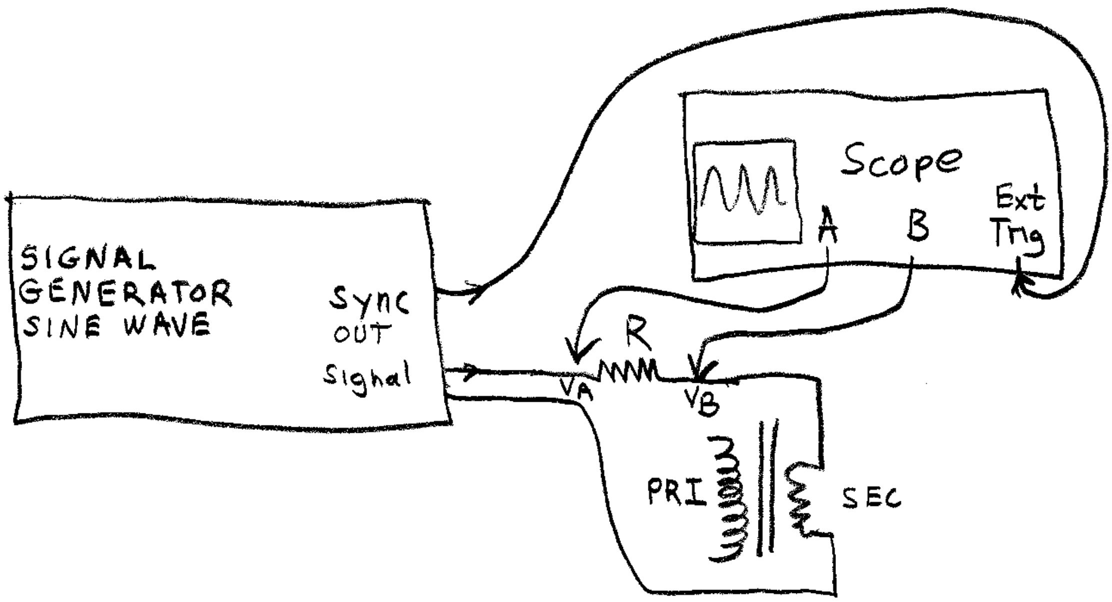

Figure 2: Measuring secondary impedance

We first analyze the circuit and then we can apply the results. We begin by noting that the current through the transformer winding is the vector voltage across resistor, R, divided by R.

$$I = \frac{(V_A - V_B*\cos(\theta)) - jV_B*\sin(\theta)}{R} \qquad \text{Eq. 5}$$

The impedance of the secondary winding is the voltage across the winding divided by the current through it.

$$Zs = \frac{R * (V_B * \cos(\theta) + jVB * \sin(\theta))}{(V_A - V_B*\cos(\theta)) - jV_B*\sin(\theta)} = \frac{R * (\cos(\theta) + j*\sin(\theta))}{(V_A/V_B - \cos(\theta)) - j*\sin(\theta)} \qquad \text{Eq. 6}$$

Multiply both numerator and denominator by the complex conjugate of the denominator to obtain

$$Zs = R * \frac{\cos(\theta)*(V_A/V_B - \cos(\theta)) - \sin^2(\theta) + j*\sin(\theta)*[V_A/V_B - \cos(\theta) + \cos(\theta)]}{(V_A/V_B - \cos(\theta))^2 + \sin(\theta)^2} \qquad \text{Eq. 7a}$$

After some simplification we have

$$Zs = R * \frac{\cos(\theta) - (V_B/V_A) + j*\sin(\theta)}{(V_A/V_B) - 2*\cos(\theta) + (V_B/V_A)} \qquad \text{Eq. 7b}$$

Note that Eq. 7b is of the form $Zs = Rs + j*X_L$ where Rs is the secondary AC resistance at the test frequency and X_L is the secondary inductive reactance at the test frequency. Note that the first and last terms of the denominator are reciprocals. For a good transformer, Rs should be much less than X_L.

Although all we really need to know is the inductive reactance, while we are at this point we can calculate the inductance of the secondary in henries as

$$Ls = X_L / (2 * \pi * 1000) \qquad \text{Eq. 8}$$

Given F_{LOW} in Hz, the lowest audio frequency of interest and using the rough rule-of-thumb XL being three times the resistive impedance at the lowest frequency, the nominal load resistance for the transformer is

$$R_{Lnom} = 0.33 * XL * (F_{LOW} / 1000) \qquad \text{Eq. 9}$$

For crystal radio use, choose an F_{LOW} of around 100 to 200 Hz. The transformer will work well with any load resistance between about one half and twice this value.

Measuring the primary impedance

Refer to the circuit in Figure 3 and connect a load resistor, R_L, across the secondary equal to the load impedance of the headphones or speaker that will be used. This assumes that your load impedance is roughly within a factor of two of R_{Lnom} calculated above.

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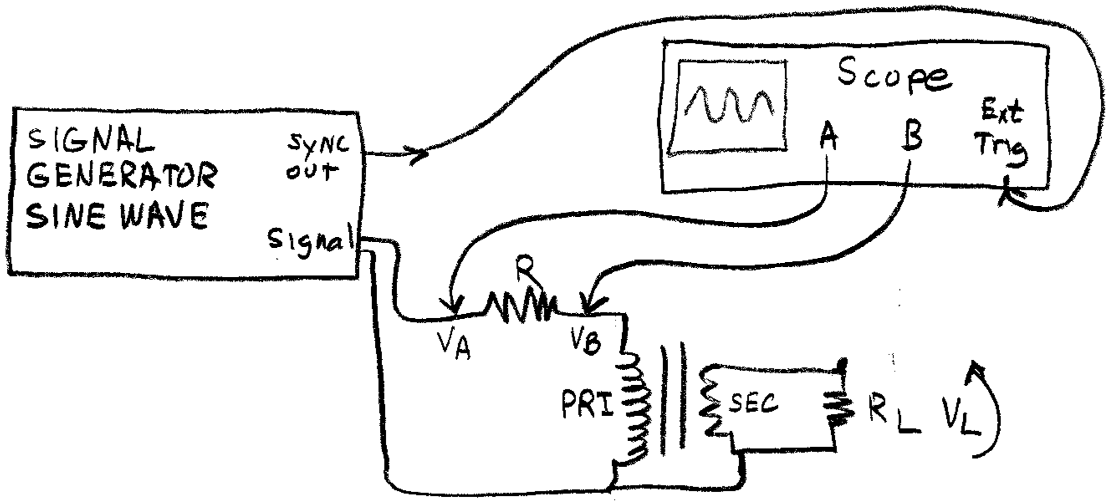

Figure 3: Measuring primary impedance

Set the signal generator to a 1 KHz sine wave and adjust the output amplitude for about 0.6 volts peak-peak as measured on the oscilloscope. Adjust R until the magnitude of the voltage on channel 'B' of the scope is roughly half the value of channel 'A'. This provides the best resolution of the measurement. In measuring V_A and V_B you should note if the phase angle between them is close to zero. If not then there is a problem and the following calculations will be in error.

$$V_B/V_A = R_p / (R + R_p) = (R_p/R) / (1 + R_p/R) \quad \text{Eq. 10}$$

Solving for R_p gives

$$R_p = R * (V_B/V_A) / (1 – V_B/V_A) \quad \text{Eq. 11}$$

Measuring the transformer power loss

Without changing anything in the previous setup, measure the peak-peak voltage across RL using scope channel 'A' – you will probably have to adjust the vertical scale amplitude setting to measure the smaller signal. Call this voltage V_{Lpp}

Note that the power delivered to R_L is

$$P_{RL} = V_{Lpp}^2 / (8*R_L) \quad \text{Eq. 12}$$

Note that the power delivered to the primary of the transformer is

$$P_{Rp} = V_{Bpp}2 / (8*R_p) \quad \text{Eq. 13}$$

If things are going correctly, then P_{RL} will be a little less than P_{Rp} because the transformer has power loss. The loss in db is

$$P_{loss} = 10 * \log_{10}(P_{RL}/P_{Rp}) \quad \text{Eq. 14}$$

Use the following chart as a guide to the suitability of your transformer for a crystal radio. Keep in mind that even an "awful" transformer is probably better than nothing so use that until you can obtain a better one.

Ploss	Quality
<1 dB	Excellent – few audio transformers will achieve this
1 – 2 dB	Good – most audio transformers will achieve this
2 – 3 dB	Fair
3 – 5 dB	Poor
>5 dB	Awful – either a mistake or this transformer is not meant for audio

Measuring the transformer frequency response

The frequency response achieved by a transformer is influenced by the external circuit it is connected to. The most accurate way to measure the response is to use the actual circuit. This experiment shows a basic circuit that provides good results and forms the basis for comparing transformers.

Connect the transformer using the circuit in Figure 4. The load resistor is either that of your headphones or speaker. Connect an AC voltmeter or scope across the load resistor. Set the output of the signal generator to be a 1 kHz sine wave with an amplitude such that the voltage across the load resistor is easy to measure (for oscilloscope use a suggested value is 0.6 Vpp and the scope set on 0.1 volt per division). Measure this voltage and refer to it as Vref. Do not adjust the amplitude of the generator after this point – only adjust the frequency.

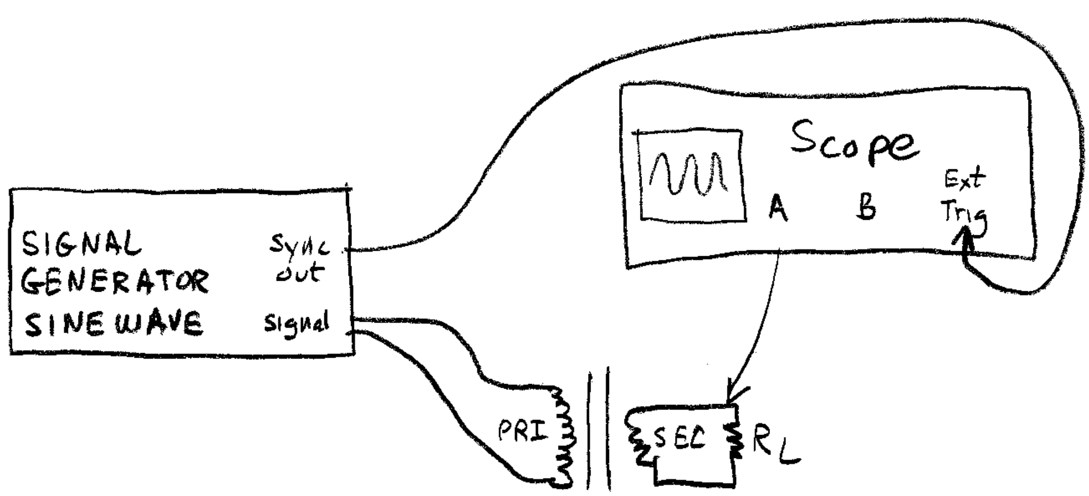

Figure 4: Measuring frequency response

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Measure the amplitude across the load resistor at each of the following frequencies. The range shown is all you need to be concerned with for a crystal radio but if you are interested you might want to extend the range of test frequencies using the same coarse sequence of test frequencies. Compute the response (relative to 1 kHz) for each frequency using the following equation.

Response = 20 * $\log_{10}$(Vmeasured / Vref) Eq. 15

Test Frequency	Measured Amplitude	Computed Response	
100	________	________	dB
150	________	________	
200	________	________	
300	________	________	
450	________	________	
650	________	________	
1000	________	0.0	dB – by definition this point is 0 dB
1500	________	________	
2000	________	________	
3000	________	________	
4500	________	________	

If the response data is plotted the curve should be generally smooth and will probably show drop-off at each end of the spectrum. It would not be unusual for the response at 100 Hz to be down by three or more dB. The response at 4500 Hz may be down by around one dB. You might see a broad dB or more increase in the response at some region (a sharp deviation at a single frequency is likely to be the result of an error). The standard method for rating the frequency response of a transformer is the band over which the response is within a +-3 dB variation from some norm. That coarse specification tends to hide the limited bandwidth that transformers can achieve.

Your transformer is suitable for a crystal radio if its frequency response over the 300 to 3,000 Hz range does not vary more than roughly 6 dB.