Crystal Radio Engineering
Designing an air-core Inductor

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Introduction

This chapter describes the mathematical process for designing an air-core inductor comprised of a single layer solenoid winding over a rigid coil form. Although the development of the mathematics is a bit complicated the final result is simple to apply. For practical reasons, this chapter will make use of English rather than metric units.

A well designed and constructed air-core coil has better performance than those with ferrite cores. Ferrite acts as a flux multiplier and has the advantage that the physical size of the inductor can be reduced. That is very important for small radios and the chief reason ferrite is used. The price paid for small size is loss of performance but that loss is generally negligible in active radios. The loss is not bad for crystal radio performance and many good crystal radios have been built using ferrite core inductors. But ferrite is not required. Purists correctly argue that the coil should be air core as that is how early radios were built. A mediocre ferrite core inductor will work considerably better than a poorly designed air-core one and that has probably led to the popularity of ferrite as the process for designing good air-core inductors is not widely known. This chapter reveals those secrets.

Analytic equation

The classic equation (which you can find in any book or article about winding inductors) for calculating the inductance of a given single layer coil is (Reference 2):

$$L = \frac{r^2 * n^2}{9*r + 10*l} \qquad \text{Eq. 1}$$

where:
L = inductance in microhenries
r = coil radius in inches (center of coil to center of conductor)
n = number of turns
l = coil length in inches (center of starting turn to center of ending turn)

This equation is generally accurate to around one percent for inductors of common dimensions. It is more convenient to work with coil diameter and Equation 1 can be written as:

$$L = \frac{d^2 * n^2}{18*d + 40*l} \qquad \text{Eq. 2}$$

where d is the coil diameter in inches (center of conductor to center of conductor)
Example: What is the inductance of a coil has a diameter of 2.5 inches, a length of 2.33 inches, and has 72 turns?

$$L = \frac{2.5*2.5*72*72}{18*2.5 + 40*2.33} = 234 \text{ uH}$$

Development of design equations

Equations 1 and 2 are fine for determining the inductance of an existing coil but are very awkward to apply to the design of a desired coil as there are many variables. Any time there are a multitude of variables then the possibility of optimum combinations or relations should be explored. In the following development the number of variables is reduced by finding relations to known constants. We first replace the coil length by a factor that relates it to the diameter.

$$l = k*d \qquad \text{Eq. 3}$$

where
l = coil length in inches
k = a dimensionless constant
d = coil diameter in inches as before

Substituting Equation 3 into Equation 2 gives:

$$L = \frac{d^2*n^2}{18*d + 40*k*d}$$

which reduces to

$$L = \frac{d*n^2}{18 + 40*k} \qquad \text{Eq. 4}$$

It can be shown that the value of k that minimizes the length of wire to wind the coil is 0.450. However, other research indicates (see Reference 1) that the value of k that minimizes coil losses is approximately 0.96 even though that value uses about twenty percent more wire. Factors contributing to coil losses include:

- Ohmic losses in the wire including skin-effect
- Dielectric losses in the coil form and nearby materials
- Dielectric losses in the insulation around the wire

- Induction losses in nearby materials

There are also losses caused by adjacent turns being too close together. It has been found (see Reference 1) that the optimum spacing (wire center to wire center) of adjacent turns is between about 1.3 to 2.0 times the diameter of the conductor. Coils for crystal radios are commonly wound using what is known as magnet wire (thin enamel insulation) and the turns are tightly wound next to each other corresponding to a spacing factor slightly greater than 1.0 (the thin insulation is of finite thickness). Although it is less than the optimum discussed it works well.

Without some special technique (such as a lathe) it can be very difficult to manually wind a coil with controlled spacing between the turns. One easy method for achieving a spacing factor of 2.0 is to wind two wires tightly side by side at the same time and then remove one of the windings when finished. Smaller spacing factors can be achieved using a smaller diameter wire for the spacer but the difficulty of controlling two wires will increase. It might occur to someone to use a wire with a thicker insulation so that a spacing is naturally formed with a tight winding. The problem with this method is that the insulation may increase dielectric losses and become self defeating – although this may be a small issue – be sure to try it before tossing the concept. This method can work great if Teflon wire is used as that is a very low-loss material and the internal wire strands are silver plated.

Equation 4 can be used to determine the optimum coil diameter for a given inductance and wire size. We note that the coil length is the number of turns divided by t (turns per inch of the wire). We also note that the coil length has previously been related to the coil diameter by the constant, k. Thus:

$$n = k*d*t \qquad \text{Eq. 5}$$

Substituting Equation 5 into Equation 4 gives:

$$L = \frac{k^2*t^2*d^3}{18 + 40*k} \qquad \text{Eq. 6}$$

We will use 0.96 for k and t will be that of the particular wire we have available. Solving Equation 6 for the optimum diameter gives:

$$d\_optimum = 4*(L/t^2)^{1/3} \qquad \text{Eq. 7}$$

Figure 1 shows a plot of this Equation 7 for common wire sizes. In all cases the turns are close-spaced. The lower curves are for common enamel insulated magnet wire. The two upper curves are for vinyl insulated house wire which can be considered if a large diameter coil form is available. To use the curves, select the desired inductance and the wire size that will be used. Look up the optimum coil form diameter and then use the closest practical form you have to that size. The optimum is broad so do not worry about being exactly on it. Note that the true diameter is the sum of the diameter of the coil form and the diameter of the wire since by definition the coil diameter is measured between opposite centers of the wire.

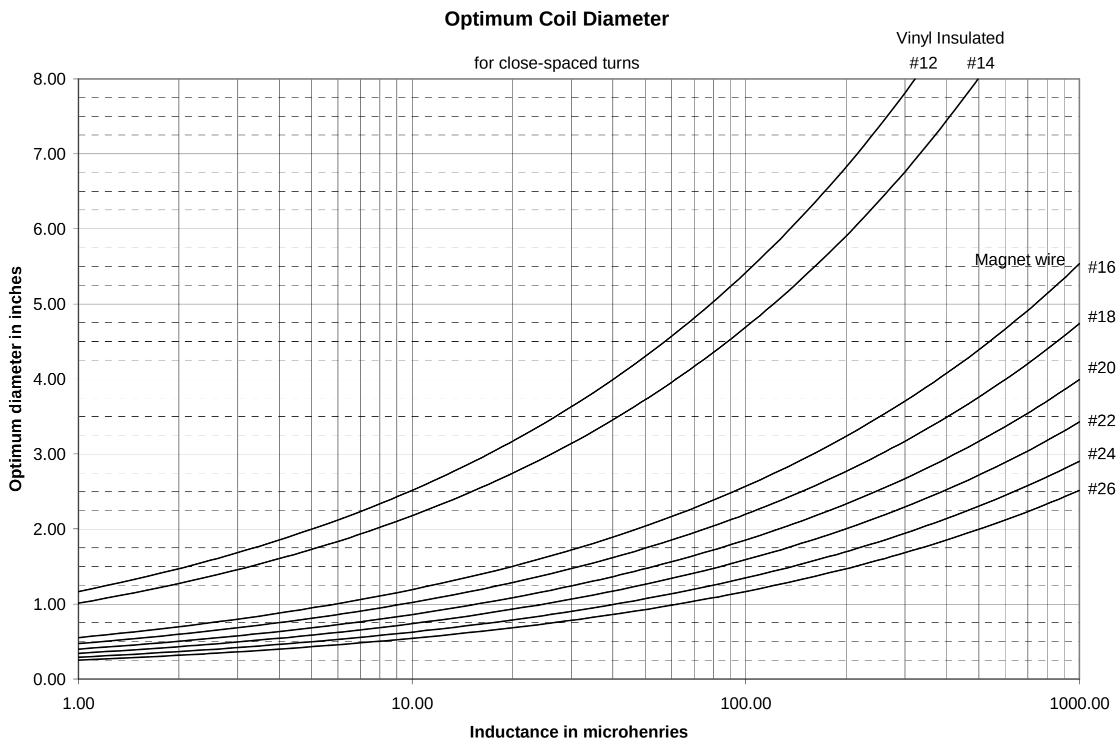

Figure 1: Optimum coil diameter

The following table provides typical values for t (turns per inch) for some common wire sizes:

Table 1: Wire data

Gauge	t	Comments
12	6.2	Vinyl insulated house wire
14	7.7	Vinyl insulated house wire
16	19	Enamel insulated magnet wire
18	24	ditto
20	31	ditto
22	39	ditto
24	50	ditto
26	62	ditto

The length of the winding will be the number of turns divided by the turns per inch of the wire. That is:

$l = n/t$ Eq. 8

We now substitute Equation 8 into Equation 2 and solve for n

$d^2*n^2 – 18*d*L – 40*(n/t)*L = 0$ Eq. 9

$t*d^2*n^2 – 40*L *n – 18*d*t*L = 0$ Eq. 10

Solving for n gives:

$$n = \frac{20*L + sqrt(400*L^2 + 18*t^2*d^3*L)}{t*d^2}$$ Eq. 11

Although a precise value (in inches) for the length of wire required can be calculated using trigonometry for a spiral, a very close value can be calculated as

$w = pi*d*n$ Eq. 12

Remember that d is the sum of the coil form diameter and the diameter of the wire. This approximation assumes that the diameter of the wire is very small in comparison to that of the coil form. Also remember to allow an extra couple of inches for connecting leads at each end of the coil.

Example: A 300 uH coil is needed. The expected Q should be over 350. What coil diameters and wire sizes could possibly meet this?

Solution: Using Figure 2 it can be seen that wire sizes #12, #14, #16, #18, and #20 could achieve the required Q. Using Figure 1 the required coil form diameters are:

Gauge	Optimum diameter
#12	7.75”
#14	6.75”
#16	3.75”
#18	3.15”
#20	2.65”

A piece of 4.5” OD PVC pipe is available and #14 electrical wire is available. From Table 1, #14 insulated wire will make about 7.7 turns per inch. Thus, the effective diameter is 4.5 plus 1/7.7 = 4.63 inches. Using Equation 11 the number of turns required is 86. Using Equation 8 the length of the winding is 11.2 inches. The length/diameter ratio is 2.4 which is a bit longer than the optimum of 0.96. The length of wire required is given by Equation 12 and is 1,251 inches. The length would have been 1,058 inches if the optimal diameter could have been used. This extra length will cause somewhat higher losses – it might still meet the desired spec though. This is about as far as I would go in rounding to an available coil form diameter.

Estimation of Inductor Q

All inductors have an equivalent series resistance loss as discussed earlier and is comprised of a number of components. We measure the quality factor or Q of the inductor by computing the ratio of inductive reactance at the frequency of interest to the series loss resistance as follows:

$$Q = \frac{X_L}{Rs} \qquad \text{Eq. 13}$$

where
Q is the dimensionless "quality" factor of the inductor
X_L is the inductive reactance in ohms at the frequency of interest
Rs is the equivalent series resistance in ohms at the frequency of interest

Note that inductive reactance, X_L, is calculated as

$$X_L = 2*\pi*F*L$$

where
F is the frequency in Hz
L is the inductance in henries

The equivalent series resistance is the net of ohmic losses including skin effect, dielectric losses in distributed capacitance and coil structure, absorption losses by nearby conducting media, magnetic losses in nearby magnetic media, etc. With care these losses can be kept small but it takes very little loss to reduce the Q of an inductor from 400 to 200. The magnitude of Rs can be measured on sophisticated impedance equipment but it is hard to calculate the effect of all factors. Figure 2 shows an estimated value of Q at 1 MHz considering typical losses assuming the coil is wound optimally and is not disturbed by nearby lossy materials. Use the figure only as a guideline as your specific results may be better or worse. The expected Q at 540 kHz will be between about 50 to 70 percent of what is shown and the expected Q at 1.6 MHz will be around 1.2 to 1.5 times that shown.

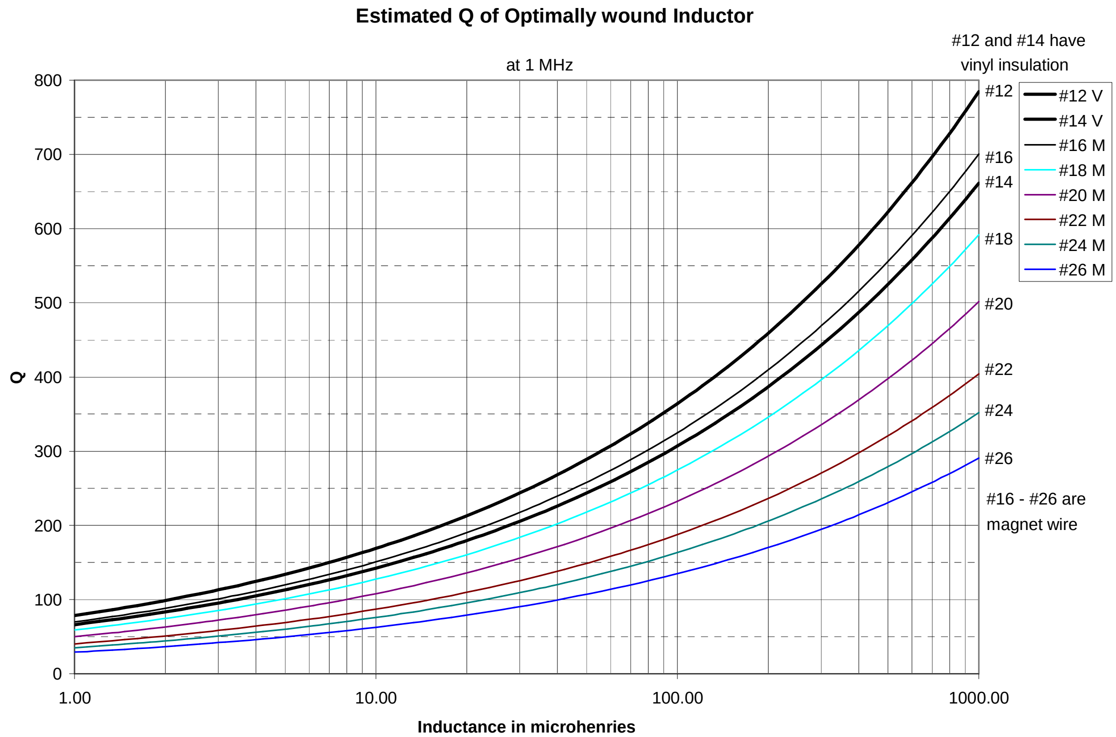

Figure 2: Estimated Q of Inductor

The Q we obtain from Equation 11 is for the unloaded coil (i.e. antenna and crystal detector not connected). The net loaded Q will typically be significantly smaller but ideally (as discussed in another chapter) would be in the general range of one hundred. Thus, we would like to start with an unloaded Q of several hundred. As can be seen in Figure 2 the Q of the inductor can be made higher by using larger diameter wire. From Figure 1 this also means using a large diameter coil form. This is a very important conclusion – high Q coils need to be physically large.

Type of wire

The only material to consider for the wire is copper. A variety of styles of copper wire is readily available. The most basic choice is between solid or stranded. Although a variety of arguments can be made for and against each, in practical terms you will not notice any difference in performance although one or the other may have physical advantages for your particular construction method. Avoid wires that are plated as those will have higher losses since skin-effect will cause most of the conduction to be in the plating which has higher resistance than copper. Avoid wires with rubber or cheap plastic insulations as dielectric losses will be higher. An exception is silver plated Teflon wire as that has the best conductivity and the lowest dielectric losses – but it is expensive.

For use in low to medium frequency inductors there is a special wire called Litzengrad or just Litz for short. It is designed to minimize skin-effect losses and is made by assembling many strands of enamel insulated magnet wire together to form a wire that has a large surface area. Litz wire is not easy to find and tends to be expensive. If you are going to use Litz wire then make sure that other losses as previously discussed are minimized. Otherwise Litz wire will make little if any difference and will be wasted effort and expense.

Avoid belief in a variety of myths about skin-effect. Although it is true that skin-effect is more severe on large diameter conductors, a larger diameter still conducts better than a smaller diameter at any frequency. This can be seen in Figure 3 which shows the frequency dependence of the resistance per meter factor of some common wire sizes.

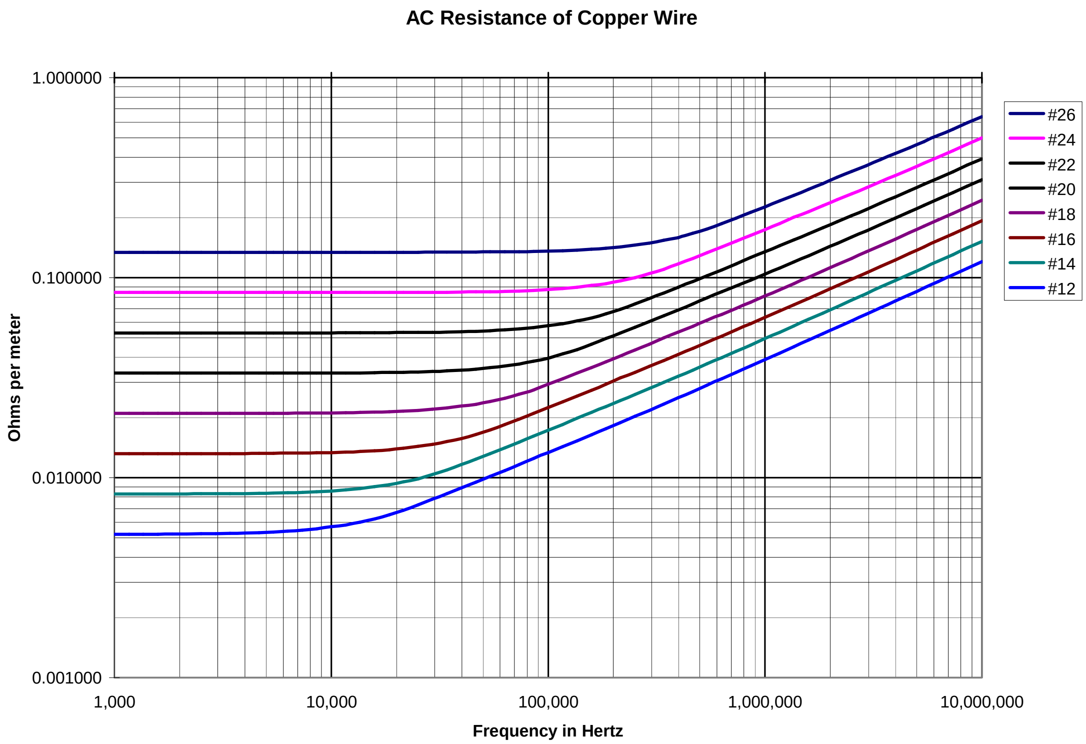

Figure 3: AC Resistance of Copper Wire

Coil Forms

From a loss standpoint air is the best coil form material there is. The obvious problem is that air has no structural strength. However, there are methods used by commercial inductor companies that employ a minimal structure so that the coil form is around 99 percent air. Manually, you can achieve the effect by first winding the coil using large diameter solid copper wire (i.e. #18, #16, #14, etc.) on a rigid coil form and then carefully sliding the winding off of the form. The stiff wire will retain the shape and you can easily space the turns to the optimal discussed previously. You will need a few supports to keep the whole thing from being too loose.

A popular coil form is some kind of cardboard tube that you have salvaged from a variety of sources such as used for paper towels or shipping tubes. These are great if you are using small diameter wire as small wire will not self support. Plastic pipe is another material you might consider. None of these materials are made with any consideration about high frequency dielectric losses but because only a small amount of material is used the losses are probably minimal.

Wood is a convenient coil form and has low losses if very dry. Common sizes that have been used are 2x2, 4x4, or a pair of 2x4 combined to make 4x4. Round dowel rods may also be used but their diameters are often much less than optimum. When using a square coil form there is a logical question about how that affects the inductance calculations. A simplistic (but good) answer is to use an effective circle diameter that has the same area as the square form since area is a strong factor in inductance. Losses with a square form will be somewhat higher than for a circular form. Rectangular forms (such as a single 2x4) have even higher losses in comparison – it takes more wire to encompass a given area.

Winding the coil

Counting turns is a tedious and error prone task. It is much simpler to cut the length of wire needed and then wind that until finished. The resulting turns count will be very close if not exact. Wire is springy and will jump off the form in a tangled mess if not restrained. Start by securing the wire at one end of the form and have a means for easily (preferably with one hand) securing the opposite end when you finish. It is tempting to use some kind of adhesive tape and that will work if you are careful and understand what you are doing. The forces will build and the tape may give way which will result in a frustrating mess of tangled wire. Make sure the tape can not slip. A good way to secure the ends is to first drill a hole in the tube at the starting and end points. Then feed the starting end through the starting hole and bend the wire such that it naturally resists tension and secure the wire with tape. Stuff the loose end length inside the tube so it is out of the way while winding.

It is best to wind the coil by hand as the set up for using a lathe is not worth the trouble for a single coil. There are a number of “poor man’s” lathes such as a power drill that have been used but I do not recommend that as you are more likely to make a mess or cause injury than you are to wind a coil. It only takes a couple of minutes to wind a coil by hand so take the time to think what you are doing. It is important to keep the winding tight at all times. The wire will spring off if it ever gets loose. You will very likely have some fractional turn as a result of your calculations. I recommend that you round that to the nearest integer as it is not worth the trouble of making measurements to stop at a specific fractional turn.

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References:

1. Electronic and Radio Engineering, fourth edition, Frederick Emmons Terman, McGraw-Hill Book Company, 1955, pages 30 – 33.

2. The Radio Amateur's Handbook, 44th edition, 1967, The American Radio Relay League, Newington, Conn., page 26