Crystal Radio Engineering
Resonant Circuit

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The resonant circuit plays a very important role in a radio receiver. Its primary purpose is to be a tunable narrow band-pass filter for selecting a desired station while rejecting undesired stations. An important secondary purpose is to be a means for transforming the low impedance antenna up to the high impedance detector.

A resonant circuit is comprised of an inductor and a capacitor, one or both of which may be variable. Energy flows back and forth between the inductor and capacitor at the resonant frequency. Energy in the inductor is stored in a magnetic field and energy in the capacitor is stored in an electric field. If there were no losses, this cyclic process could continue forever. In reality there is some loss each time the energy moves. This loss is expressed as a resistance. A common term used to account for this loss is Q which is the inverse of the loss. Thus, a high-Q resonance has very low loss and the cyclic process can continue for many cycles. A common example is a bell which continues to ring long after being hit.

Figure 1 shows the frequency response of a resonant circuit at 1 MHz for a range of Q.

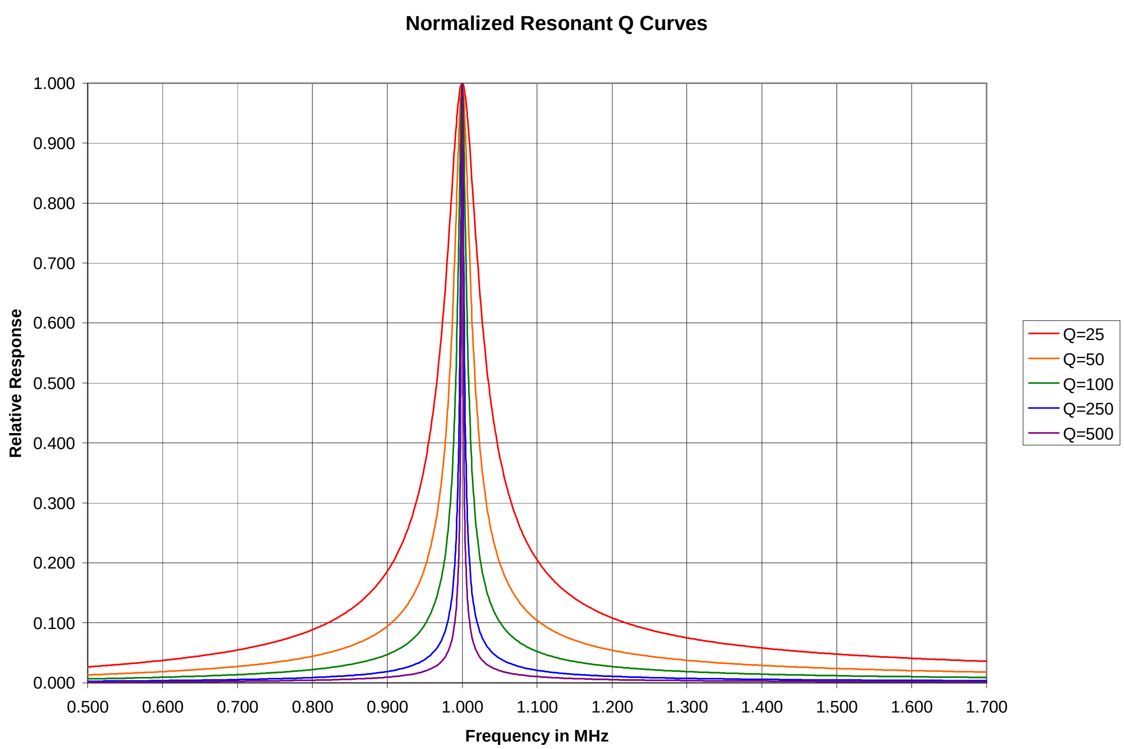

Figure 1: Normalized resonance curves at 1 MHz

Crystal Radio Engineering
Resonant Circuit

Figure 2 shows a zoomed in version of Figure 1.

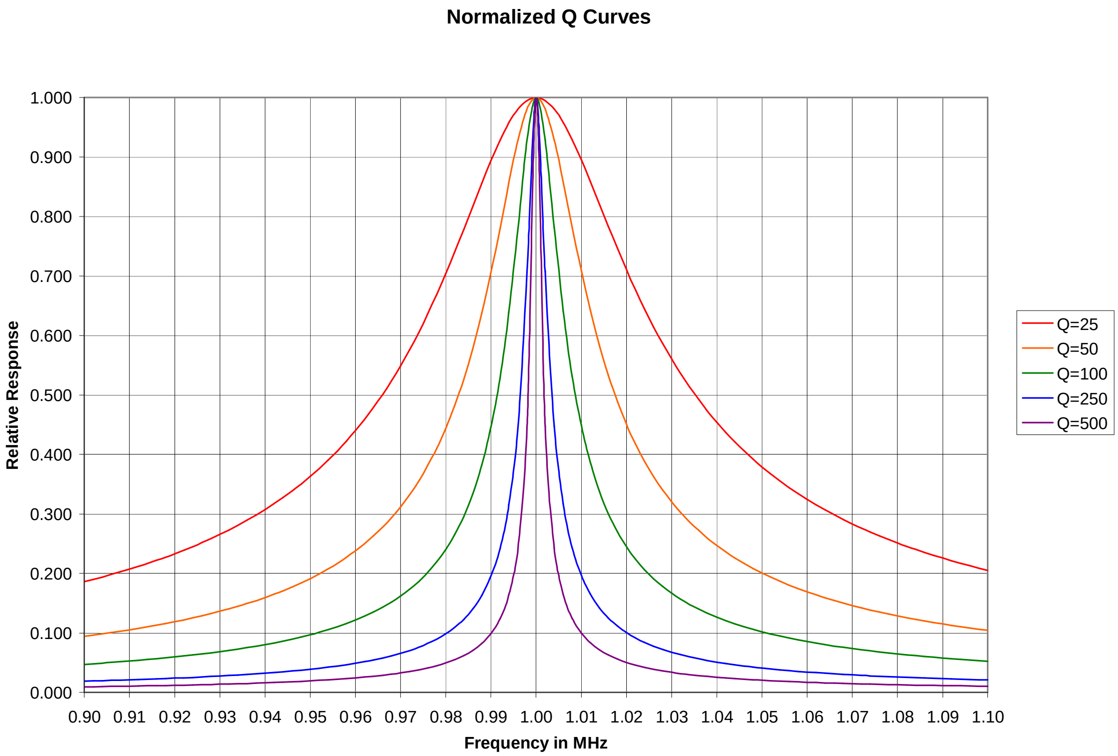

Figure 2: Normalized resonance curves at 1 MHz

Note that the Q=100 curve has a band-pass of 10 kHz which is about the minimum that can be used to recover the entire double-sideband signal.

Tuning capacitor

Generally, when the capacitor is variable, the inductor is fixed and visa-versa. Mechanical variable capacitors (often 10 – 365 pF or similar) were very popular in older times but are more difficult to find in modern times. A modern substitute can be made using one or more rotary switches as will be illustrated later.

The well know equation for the resonant frequency of an inductor and capacitor is

$$F = \frac{1}{2 * \pi * \mathrm{sqrt}(L * C)} \qquad \text{Eq. 1}$$

where
F = resonant frequency in Hz
L = inductance in henries
C = capacitance in farads

Crystal Radio Engineering
Resonant Circuit

For a given inductance the required value of capacitance can be determined from

$$C = \frac{1}{(2 * \pi * F)^2 * L} \qquad \text{Eq. 2}$$

The required tuning range, Cmax / Cmin, of a variable capacitor can be determined using Equation 2 as follows

$$\frac{Cmax}{Cmin} = \frac{\dfrac{1}{(2 * \pi * Fmin)^2 * L}}{\dfrac{1}{(2 * \pi * Fmax)^2 * L}} = \frac{Fmax^2}{Fmin^2} \qquad \text{Eq. 3}$$

For the AM broadcast band Fmax is generally 1.7 MHz and Fmin is 0.54 MHz. Thus, the ratio of Cmax to Cmin should be around 10.

Tuning inductor

The classic ancient method of making a variable inductor was to provide a movable wiper to connect to individual turns of the winding. Modern methods use a movable ferrite rod to vary the inductance. Similarly to the development of Equation 3, the ratio of Lmax to Lmin is the square of the desired frequency ratio and works out to be around 10 for the AM broadcast band.

Discussion of Q

Q can be defined in several ways and each way is compatible with the others as shown below.

Q = resonant frequency / $\text{bandwidth}_{3\ db}$ Eq. 4

Q = Rshunt / reactance Eq. 5

Q = reactance / Rseries Eq. 6

Inductors and capacitors have internal losses which appear as a resistance load either in series or shunt with the component. Losses in picofarrad capacitors used at broadcast frequencies tend to be very small and thus these capacitors have a high Q (typically many hundreds) or as it is more commonly referred to in capacitors, low dissipation (the reciprocal of Q). Inductors have the dominant loss. This loss is commonly the series resistance of the wire (including skin-effect) but can also include losses in any magnetic medium used in the inductor. For good crystal radios we need inductors to have a Q in the hundreds.

Resonant circuits are not used in isolation. The antenna is coupled to the resonator as well as the diode detector. Both of these represent a resistive load which lowers the Q of resonance. This is referred to as the loaded Q. That may sound bad but it is actually fine. The loads represent power that we are interested in. We prefer that the resonator have low losses or high-Q. That permits the maximum transfer of power from the antenna to the detector.

Since any resistive load such as the antenna radiation resistance or the detector resistance lowers the net Q (i.e. Qloaded) of resonance, it is important to start with a high unloaded Q known as Qu. The Qu of an LC resonant circuit is primarily determined by the Q of the inductor as its losses tend to dominate. Typical values for the inductor Q range from around 50 to over 500. If the tuning capacitor is of very good quality (i.e. air or silver mica insulation, etc.) then its Q will typically be in the 500 to over 2000 range.

Although we like for the unloaded Q of the resonator to be as high as practical for low losses, it is desirable for the loaded Q to be no more than about 50 so that the bandwidth is not too narrow. An excessively narrow bandwidth makes tuning very sharp and also distorts the audio. The bandwidth needs to be at least 10 kHz with 20 to 50 kHz being common. The following table illustrates the maximum, typical, and minimum loaded Q values to use across the AM broadcast band. Keep in mind that the unloaded Q of resonance should be significantly higher – preferably in the hundreds. The maximum Q value is for the minimum bandwidth of 10 kHz. The typical Q is a good value to try to achieve although realistically it is likely to be lower. The minimum Q has a fairly wide bandwidth and will not be able to separate stations well.

Frequency	**QLmax**	**QLtyp**	**QLmin**
0.5 MHz	50	25	10
1.0 MHz	100	50	20
1.7 MHz	170	85	34

Table 1: Practical ranges for Q of resonance in the AM broadcast band

The resonator has an effective resistance across it that represents coil losses. This resistance will be referred to as RQ. Ideally, RQ is infinity (i.e. no losses) but realistic values are typically in the many tens to hundreds of thousands of ohms. There are two loads on the resonator – one is effective source resistance, RS, of the matching network to the antenna and the other is the effective load resistance, RL, of the diode detector and audio transducer. For maximum power transfer from the antenna to the audio transducer the source and load resistances should be equal.

How to determine the required inductance

Although there are an infinite number of combinations of inductance and capacitance that will resonate at a desired frequency there is only a limited range of practical values that can be used. An excellent question to ask is if there is an optimum inductance. If there is then that is what we will use. There is not a specific answer to that question other than there is an identifiable range of inductance that provides the best overall results. We will determine the extremes starting with the minimum practical value that will result in the highest allowable loaded Q followed by calculating the maximum practical value that will result in the lowest acceptable loaded Q. Any practical value of inductance between these two extremes can be used.

Determination of minimum practical inductance: As discussed previously, the highest loaded Q of resonance that is practical to use is such that the 3 dB bandwidth is about 10 kHz. This permits the full double sideband width to be detected. In extreme cases where we are willing to forfeit some audio fidelity to achieve better selectivity the bandwidth can be reduced to about 6 kHz – but we are not going to consider that case here. At 1 MHz (the rough center of the AM broadcast band), a 10 kHz bandwidth occurs with a loaded Q of 100. The antenna and detector circuits may be operating from taps on the inductor for the purpose of impedance matching (the method to design the taps is discussed in another chapter). The impedance at each tap can be transformed to an equivalent impedance across the entire coil. We will assume that the antenna circuit and detector circuit are impedance matched via this tapping process (this provides the much needed maximum power transfer from the antenna to the detector). Thus, for the impedance matched condition the antenna and detector impedance will transform to the identical impedance across the inductor. We will refer to the net parallel impedance of these two across the coil as Rsignal. It was discussed previously that losses in the inductor can be represented by an effective resistance across the entire coil. We will refer to this loss resistance across the coil as Rloss. The parallel combination of Rsignal and Rloss is the net resistance across a lossless inductor. We will refer to this net resistance as Rshunt. From Equation 5 we can write

XL = Rshunt / loaded Q   Eq. 6

Thus, using a frequency of 1 MHz

Lmin = XL / (2 * π * 1 MHz) = Rshunt / (2 * π * 1 MHz * loaded Q)   Eq. 7

The detector impedance is typically in the 2,000 to 50,000 ohm range. With antenna matching the net resistance, Rsignal, will be half this range as discussed above. Table 2 shows a summary calculation for the minimum value of inductance to use to achieve a loaded Q of 100 for various loads either directly or transformed across the coil. The Q of 1,000,000 row represents essentially infinite Q and the Q of 1,000 is not normally attainable and are shown for reference only.

Crystal Radio Engineering
Resonant Circuit

1,000,000	**Hz, resonant frequency**					
10,000	**Hz, BW**		**100.0**	**Qloaded**		
	Rdetector transformed across entire inductance					
Qunloaded	**2K**	**5K**	**10K**	**20K**	**50K**	**100K**
1,000,000	1.6E-6	4.0E-6	8.0E-6	15.9E-6	39.8E-6	79.6E-6
1,000	1.4E-6	3.6E-6	7.2E-6	14.3E-6	35.8E-6	71.6E-6
500	1.3E-6	3.2E-6	6.4E-6	12.7E-6	31.8E-6	63.7E-6
250	954.9E-9	2.4E-6	4.8E-6	9.5E-6	23.9E-6	47.7E-6
125	318.3E-9	795.8E-9	1.6E-6	3.2E-6	8.0E-6	15.9E-6

Table 2: Minimum Inductance in Henries

All of the inductances in Table 2 are small – some very small. But our target was the absolute minimum inductance that could be used. The details would be a chapter in of itself but I can tell you that the practical absolute minimum inductance that is useful for a resonator in the AM broadcast band is around 40 microhenries as it is a challenge to obtain a high Qu for inductances less than this in that frequency range. Thus, only the last column of Table 2 is useful.

Determination of maximum practical inductance: This calculation is done the same way as before except that the lowest value of loaded Q is used. That results in a higher inductance. Table 3 is a summary calculation. Again, the top two rows are for reference only.

1,000,000	**Hz, resonant frequency**					
50,000	**Hz, BW**		**20.0**	**Qloaded**		
	Rdetector transformed across entire inductance					
Qunloaded	**2K**	**5K**	**10K**	**20K**	**50K**	**100K**
1,000,000	8.0E-6	19.9E-6	39.8E-6	79.6E-6	198.9E-6	397.9E-6
1,000	7.8E-6	19.5E-6	39.0E-6	78.0E-6	195.0E-6	389.9E-6
500	7.6E-6	19.1E-6	38.2E-6	76.4E-6	191.0E-6	382.0E-6
250	7.3E-6	18.3E-6	36.6E-6	73.2E-6	183.0E-6	366.1E-6
125	6.7E-6	16.7E-6	33.4E-6	66.8E-6	167.1E-6	334.2E-6

Table 2: Maximum inductance in Henries

Conclusions: The conclusions from studying Tables 2 and 3 are that low values of inductance are needed for low impedance circuits and that high values of inductance are needed for high impedance circuits. Typical values of inductance used for the resonant circuit in crystal radios ranges from around 100 microhenries up to around 700 microhenries with more common values in the 200 to 400 microhenry range.

We can work Equation 7 backwards to determine a good value for the net shunt resistance across the coil at 1 MHz as follows.

$$\text{Rshunt} = 2 * \pi * 1\text{ MHz} * \text{loaded Q} * L \qquad \text{Eq. 8}$$

As an example, if we have a 240 uH coil and we want the loaded Q to be 20 at 1 MHz then the total shunt load should be 30,000 ohms. Assuming coil losses are small this means that the detector impedance should be 60,000 ohms and that the antenna impedance should be transformed (via a tap near the ground end of the coil) up to 60,000 ohms. This particular scenario is a practical one that can be built.

Concluding remarks

Although it is discussed in other chapters it should be mentioned here that we generally want to make the detector impedance as high as possible in order that we can use the highest inductance practical. This results in the maximum signal voltage applied to the detector thus lowering the losses in the detector circuit.

A significant factor to be aware of is stray capacitance across the inductor. This capacitance results from the natural physics of the winding (conductors separated by insulation). This stray capacitance is typically in the 10 to 50 pF range depending on how the inductor is wound and is in shunt with the tuning capacitance. The effect is to limit the upper frequency that the inductor can be tuned too. The effect is worse in large value inductors and that typically limits the maximum practical inductance to something less than 1 millihenry. There are some advanced winding methods that minimize stray capacitance but those are beyond the scope of this chapter.