BJT AC Analysis

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Introduction

This note will discuss AC analysis using the beta, re transistor model shown in Figure 1 for the three types of amplifiers: common-emitter, common-base, and common-collector. For each type of amplifier the goal is to determine the input resistance, r_{IN}, output resistance, r_O, and voltage gain. The voltage gain may be either the unloaded stage gain, the loaded stage gain, or the net voltage gain depending on what is needed. The analysis is simplified by first developing general equations for the AC terminal resistances of the transistor and then using this result to compute the input and output resistances of each of the three amplifier types.

Before proceeding with AC analysis, we must have values for beta and r_e. We nominally use a beta of 100 unless we want to know the specific characteristics over a specific range of beta. The dynamic resistance of the forward biased base-emitter junction is given by

$$r_e = V_T / I_E \qquad \text{Eq. 1}$$

where V_T is the thermal voltage. We nominally use a V_T of 0.026 Volts but if specific AC characteristics for a particular temperature are needed, then the V_T for that temperature is used instead.

AC terminal resistances

The first step in AC analysis is to develop equations that give us the AC resistance looking into each transistor terminal. The result of these calculations will enable us to easily calculate the input and output resistances of transistor amplifiers including the bias circuit. The beta, r_e model for the transistor and the standard circuit is shown in Figure 2. We define one set of AC resistances looking out of the transistor terminals and another set of AC resistances looking into the transistor terminals. Each of these is easy to calculate.

Any combination of resistances can be reduced to a single resistor. In order that we do not have to be concerned about the specific resistors in the external circuit for our analysis, the external circuit is reduced to three resistances, R_B', R_E', and R_C'. These are the external AC resistances seen looking out of the particular terminal of the transistor.

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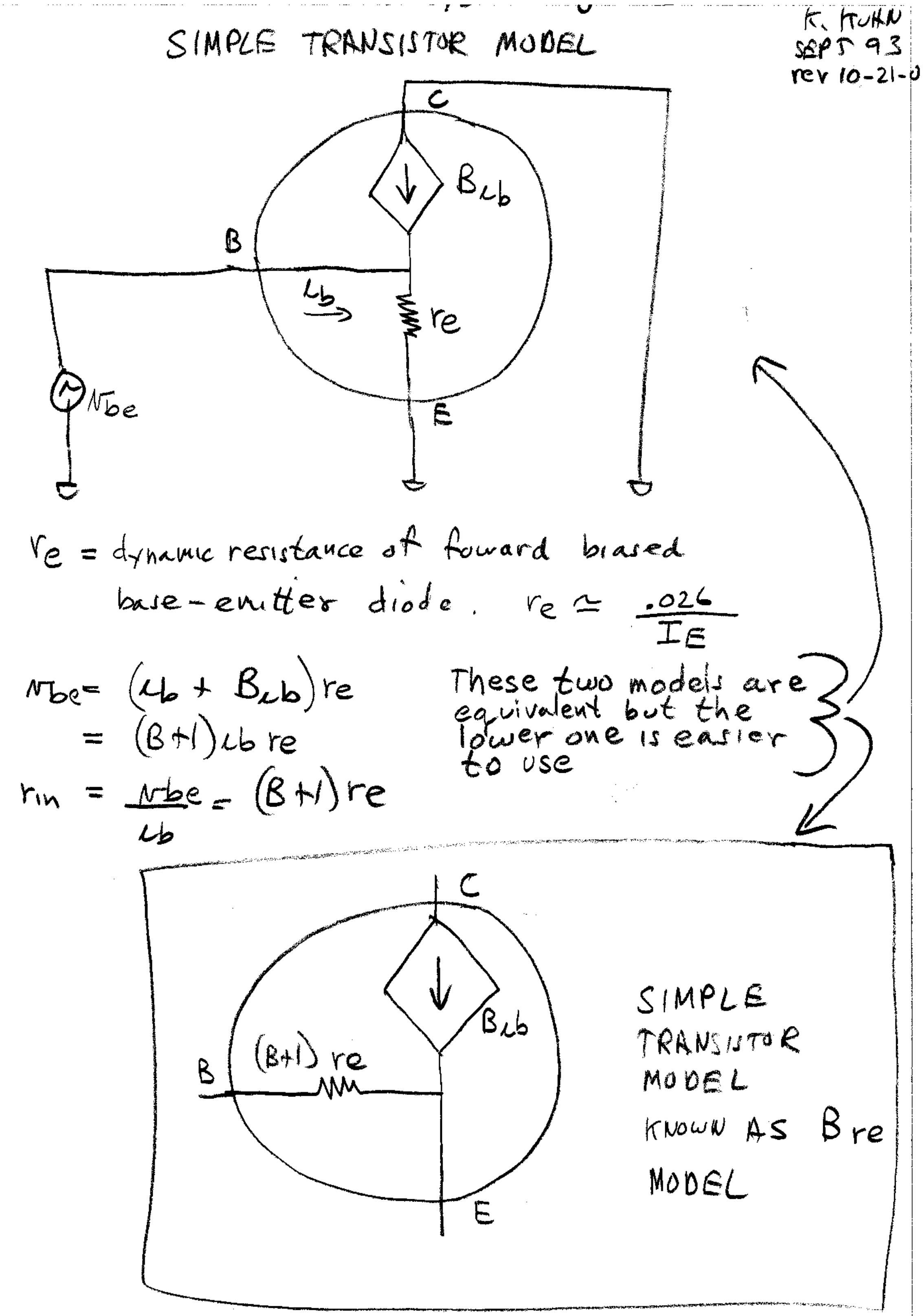

Figure 1: Simple transistor model and equivalent circuit

TERMINAL IMPEDANCES OF β, re TRANSISTOR MODEL

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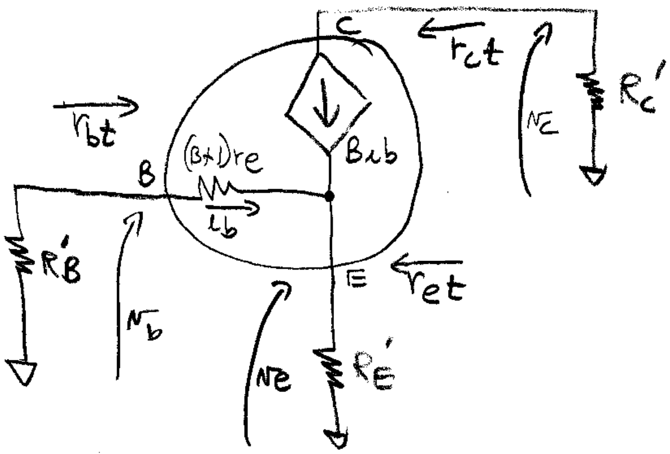

R_B', R_E', R_C' ARE NET AC RESISTANCE TO GND LOOKING OUT OF TRANSISTOR TERMINAL.

r_{bt}, r_{et}, r_{ct} ARE NET IMPEDANCE LOOKING INTO TRANSISTOR TERMINAL. NOTE THAT THE R'_ RESISTOR TO GND IS NOT INCLUDED.

THESE CALCULATIONS WILL APPLY TO THE CE, CB, AND CC CIRCUITS TO BE DISCUSSED LATER. THE MEANING OF R_B' R_E', R_C' WILL VARY DEPENDING ON WHETHER THE CIRCUIT IS CE, CB, OR CC.

NOTE THAT THE OUTPUT IMPEDANCE OF THE TERMINAL IS THE SAME AS THE INPUT IMPEDANCE.

Figure 2: Transistor impedances

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For the three types of amplifiers, R_B', R_E', and R_C' will consist of external resistance combinations different for each amplifier type. In all cases, it should be obvious how to compute these resistances. Simply imagine yourself looking out of the specific transistor terminal and seeing the net AC resistance to ground. For example, for the common-emitter amplifier, R_B' may be the parallel combination of the voltage divider bias resistors, R_{B1} and R_{B2}, also in parallel with R_S since the coupling capacitor is an AC short. R_E' may be R_E in parallel with R_{E1} for a common-emitter circuit or R_E in parallel with R_L in a common-collector circuit. R_C' may be R_C in parallel with R_L for common-emitter and common-base circuits. For the common-base circuit, R_E' is generally just R_E. The point is that no matter what the external circuit, it will be reduced to a single resistance representing the AC resistance. For this process, all coupling and bypass capacitors are treated as short circuits to AC which in fact they are at the signal frequencies we are using.

The AC resistances looking into the terminals of the transistor are called r_{bt}, r_{et}, and r_{ct}. Just to clarify the nomenclature, r_{bt} is the AC **r**esistance looking into the **b**ase **t**erminal. Note that r_{bt} does not include R_B' but is affected by R_E', r_{et} does not include R_E' but is affected by R_B', and that r_{ct} does not include R_C'.

Calculation of r_{bt}

$r_{bt} = v_b / i_b$	by Ohms' law	Eq. 2
$i_b = (v_b - v_e) / [(B + 1) * r_e]$	by inspection of the circuit	Eq. 3
$v_e = (B + 1) * i_b * R_E'$	by definition of transistor emitter current	Eq. 4
$v_b = v_e + i_b * (B + 1) * r_e$	by Kirchoff's voltage law	Eq. 5
$v_b = (B + 1) * i_b * R_E' + i_b * (B + 1) * r_e$	by substituting equation for v_e above	Eq. 6
$v_b = (B + 1) * i_b * (re + R_E')$	by combining terms	Eq. 7
$\mathbf{r_{bt} = (B + 1) * (r_e + R_E')}$	dividing by i_b produces	Eq. 8

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Calculation of r_{et}

$r_{et} = v_e / (-i_e)$	By Ohm's law. Negative because i_e is defined as positive for current leaving the emitter. We are looking in.	Eq. 9
$i_e = (B + 1) * i_b$	By definition	Eq. 10
$i_b = -v_e / [(B + 1) * r_e + R_B']$	Notice that v_e is across the sum of these two resistances	Eq. 11
$i_e = (B + 1) *(-v_e) / [(B + 1) * r_e + R_B']$	by combining the two previous equations	Eq. 12
$r_{et} = [(B + 1) * r_e + R_B'] / (B + 1)$	By solving the above equation for r_{et}	Eq. 13
$\mathbf{r_{et} = r_e + R_B' / (B + 1)}$	Simplifying the above equation	Eq. 14

Calculation of r_{ct}

$r_{ct} = v_c / i_c$	By Ohm's law	Eq. 15
$i_c = 0$	We note that $i_c = 0$ because $i_b = 0$	Eq. 16
$\mathbf{r_{ct}}$ **= infinity**	This should not be surprising since we are looking into a current source. A more complete model of the transistor would show that r_{ct} is generally between about 30,000 and 1,000,000 Ohms.	Eq. 17

The above analysis is very quick and may not be understood by all. If you have trouble understanding it then perform a two loop mesh analysis on the circuit in Figure 2 and the results should become more clear.

Calculation of Input Resistance, Output Resistance, and Voltage Gain

We are now ready to use r_{bt}, r_{et}, and r_{ct} to perform specific AC analysis for each of the three types of amplifiers. Note that the impedance calculations can now be performed by inspection. We will do an analysis for each of the three amplifier types – common-emitter, common-base, and common-collector. The circuits for these are in Figure 3.

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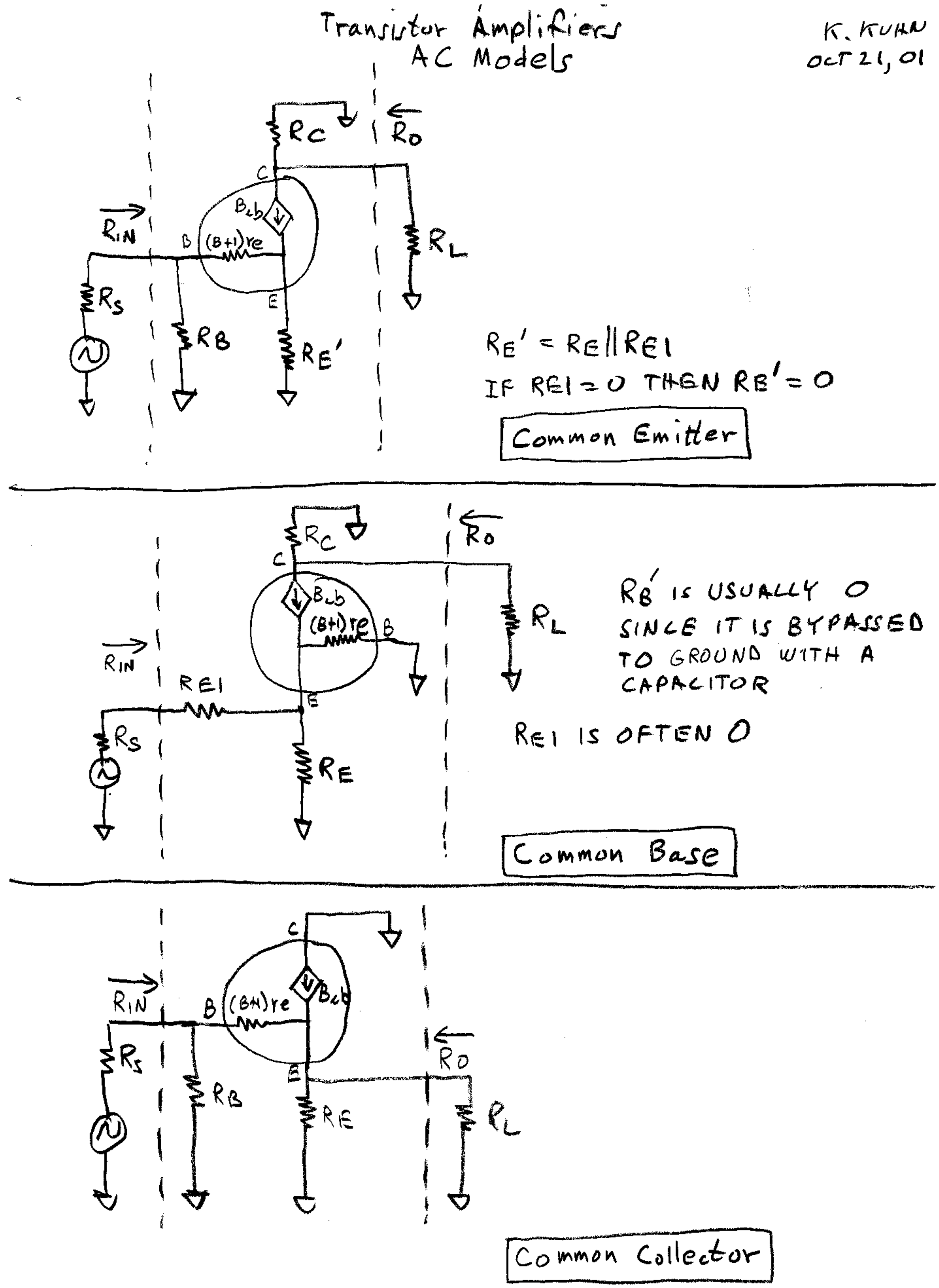

Figure 3: Transistor AC circuits

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Common-Emitter analysis

$r_{IN} = R_B \parallel r_{bt} = R_B \parallel [(B + 1) * (r_e + R_E')]$ Eq. 18

Note that $R_E' = R_E \parallel R_{E1}$. If R_{E1} is zero then R_E' is also zero. Eq. 19

$r_O = R_C \parallel r_{ct} = R_C$ since for the beta, re model, r_{ct} is taken to be infinity although real values are in the 30,000 to 1,000,000 Ohm range. Eq. 20

To find the voltage gain, we note that:

$$
\begin{aligned}
v_c &= -i_c * R_C \\
&= -B * i_b * R_C \\
&= -B * [v_b/[(B + 1) * (r_e + R_E')] * R_C
\end{aligned}
\qquad \text{Eq. 21}
$$

Noting that voltage gain is v_c/v_b, then by rearranging the above equation we can write:

$$\text{Unloaded voltage gain} = \frac{-B}{B + 1} * \frac{R_C}{r_e + R_E'} \qquad \text{Eq. 22}$$

The voltage gain is negative because the output signal is inverted from the input signal.

The output voltage division factor is $R_L / (R_O + R_L)$. Using $R_O = R_C$, the

$$\text{Loaded voltage gain} = \frac{-B}{B + 1} * \frac{R_C}{r_e + R_E'} * \frac{R_L}{R_C + R_L} \qquad \text{Eq. 23}$$

The input voltage division factor is $r_{IN} / (R_S + r_{IN})$. The net voltage gain from the source to the load is:

$$\text{Net voltage gain} = \frac{r_{IN}}{R_S + r_{IN}} * \frac{-B}{B + 1} * \frac{R_C}{r_e + R_E'} * \frac{R_L}{R_C + R_L} \qquad \text{Eq. 24}$$

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Common-Base Analysis

$r_{IN} = R_{E1} + R_E||r_{et}$
$= R_{E1} + R_E||[r_e + R_B'/(B + 1)]$ Eq. 25

R_B is often bypassed to ground using a capacitor that is a short circuit at the signal frequencies of interest. In that case R_B' is zero and then:

$r_{IN} = R_{E1} + R_E||r_e$ Eq. 26

$r_O = R_C||r_{ct} = R_C$ Eq. 27

To find the gain we note that:

$v_c = -i_c * R_C$
$= -B * i_b * R_C$
$= -B * (-i_e / (B + 1)) * R_C$
$= (B / (B + 1)) * (v_e / r_e) * R_C$ Eq. 28

$v_e = v_{in} * (r_e|| R_E) / (R_{E1} + r_e||R_E)$ Eq. 29

Noting that gain is v_c/v_{in} we can write:

$$\text{Unloaded voltage gain} = \frac{1}{R_{E1}/ (r_e||R_E) + 1} * \frac{B}{B + 1} * \frac{R_C}{r_e} \qquad \text{Eq. 30}$$

Considering the output voltage division we can write:

$$\text{Loaded voltage gain} = \frac{1}{R_{E1}/ (r_e||R_E) + 1} * \frac{B}{B + 1} * \frac{R_C}{r_e} * \frac{R_L}{R_C + R_L} \qquad \text{Eq. 31}$$

Including the input voltage division factor, the net gain from signal source to output is:

$$\text{Net voltage gain} = \frac{r_{IN}}{R_S + r_{IN}} * \frac{1}{R_{E1}/ (r_e||R_E) + 1} * \frac{B}{B + 1} * \frac{R_C}{r_e} * \frac{R_L}{R_C + R_L} \qquad \text{Eq. 32}$$

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Common-Collector analysis

$r_{IN} = R_B \parallel r_{bt}$
$\quad = R_B \parallel [(B + 1) * (r_e + R_E \| R_L)]$ Eq. 33

Note that the input resistance is influenced by the load resistance.

$r_O = R_E \| r_{et}$
$\quad = R_E \| [r_e + (R_B \| R_S)/(B + 1)]$ Eq. 34

Note that the output resistance is influenced by the source resistance.

To calculate the unloaded gain we note that:

$v_e = i_e * R_E$
$\quad = (B + 1) * i_b * R_E$ Eq. 35

$i_b = v_b / [(B + 1) * (r_e + R_E)]$ Eq. 36

Solving Equations 34 and 35 for the ratio of v_e to v_b gives:

$$\text{Unloaded voltage gain} = \frac{v_e}{v_b} = \frac{R_E}{r_e + R_E} \qquad \text{Eq. 37}$$

The influence that load impedance has on input resistance and the influence that source impedance has on output impedance leads to an algebraic mess when trying to calculate loaded gain. A better approach is to directly calculate loaded gain without using the usual output voltage division equation. By replacing R_E with $R_E \| R_L$, we can directly calculate the loaded gain following the same procedure for unloaded gain.

$v_e = i_e * R_E \| R_L$
$\quad = (B + 1) * i_b * R_E \| R_L$ Eq. 38

$i_b = v_b / [(B + 1) * (r_e + R_E \| R_L)]$ Eq. 39

$$\text{Loaded voltage gain} = \frac{v_e}{v_b} = \frac{R_E \| R_L}{r_e + R_E \| R_L} \qquad \text{Eq. 40}$$

The net loaded gain from signal source to load is:

$$\text{Net voltage gain} = \frac{r_{IN}}{R_S + r_{IN}} * \frac{R_E \| R_L}{r_e + R_E \| R_L} \qquad \text{Eq. 41}$$