Introduction to Decibels

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Introduction

The dynamic range of numerical values for some quantity can span many orders of magnitude. It can be rather cumbersome to work with some numbers in the million range and other related numbers in the single digit range. An increase of 8 in a number of 1,000,000 is a small change. An increase of 8 in a number of 10 is a large change. What is needed is a method to express numbers in such a way that puts everything in perspective.

A simple system that accomplishes this is to use the logarithm of the numbers. A very wide dynamic range is then compressed into a small numerical range. In the preceding example, 1,000,000 becomes 6 and 10 becomes 1 if base 10 (also known as common logarithms) logarithms are used. If the increase of 8 is added, the numbers become 6.0000035 and 1.255. The significance of increasing the number by 8 is now clear.

An extension to this simple system is to give the numbers a name and choose a reference level to compare numbers to since we can not take the logarithm of zero. The system we actually use is known as the decibel system and is described as follows.

The decibel System

The Bel is a logarithmic unit of intensity named for Alexander Graham Bell. Being a logarithmic unit, the Bel can not have an absolute scale (i.e. a reference of 0). It always represents a power ratio of a given power level to a reference power level. One Bel represents a power ratio of 10. A factor of 10 is a bit coarse for most work, so the decibel (one-tenth of a Bel and is abbreviated as dB) is typically used instead. It is interesting to note that the human ear can resolve a change in sound amplitude roughly on the order of one decibel. This makes it easy for people to relate to the significance of a given decibel change.

The equation to express a power ratio in decibels is

$$dB = 10 * \log_{10}(P2/P1) \qquad \text{Eq. 1}$$

The factor of 10 in the above equation is because we are using decibels instead of Bels (10 decibels equals 1 Bel). Note that there must always be two powers because dB represents a power ratio. It is not possible to convert a single power to a dB level.

In Equation 1, P2 could represent an output power and P1 could represent an input power. In this case, the power gain of some device is then represented in decibels.

Introduction to Decibels

Expressing Voltage or Current ratios in decibels

A voltage or current ratio can also be expressed in decibels although care has to be used in the interpretation since the impedance level of each voltage or current affects the result. A common error is to ignore the impedance relationship. The use of power eliminates the need to know the circuit impedances.

The equation to express a voltage ratio in decibels is

$dB = 10 * \log_{10}(V2^2/R2 / V1^2/R1)$

which simplifies to the following. Note that the exponent of 2 is factored out of the logarithm so that the multiplier becomes 20 instead of 10.

$dB = 20 * \log_{10}(V2/V1) + 10 * \log_{10}(R1/R2)$ Eq. 2

The equation to express a current ratio in decibels is derived in a similar manner and is

$dB = 20 * \log_{10}(I2/I1) + 10 * \log_{10}(R2/R1)$ Eq. 3

Note that the resistance term is different from the voltage dB expression.

Note that each expression is still a power ratio. The resistance part of Equation 2 or Equation 3 goes to zero if the two resistances are the same.

Implicit Reference

A commonly used extension of the decibel system is to incorporate the reference level in the unit. This implicit reference then lets us discuss dB levels as if they were absolute. Here are some common examples:

dBm decibels relative to a milliwatt. Thus, 0.2 Watts becomes 23 dBm. 1 microwatt becomes -30 dBm.
dBW decibels relative to one Watt. Thus, 100,000 Watts becomes 50 dBW.
dBuV decibels relative to one microvolt. Thus, 100 uV becomes 40 dBuV.
dBV decibels relative to one Volt. Thus, 1000 Volts becomes 60 dBV
dBuV/m decibels relative to one microvolt per meter. This unit is often used in radio engineering. A signal strength of 100 uV/m is represented as 40 dBuV/m.

Introduction to Decibels

Acoustical decibels

Probably the most common usage of dB with an implicit reference is dBa as applied to sound intensity (another way of saying power). The "a" refers to acoustic. Very often the "a" is erroneously omitted from the unit which then gives the false impression that dB is an absolute scale. The reference power level has been defined as a sound intensity of 1 picowatt per square meter which is roughly the threshold of human hearing. This is about the faintest sound that an average person can hear in an ideally quite room. The following chart provides some general idea of sound level. Note that the huge dynamic range is compressed into a small numerical range.

0 dBa	(1 pW/m^2)	faintest sound that can be heard
30 dBa	(1 nW/m^2)	very soft whisper
70 dBa	(10 uW/m^2)	normal talking voice
80 dBa	(100 uW/m^2)	medium loud radio
90 dBa	(1 mW/m^2)	loud radio or TV
120 dBa	(1 W/m^2)	extremely loud dance hall, pain threshold
140 dBa	(100 W/m^2)	jet engine at 100 feet

Calculations with decibels

Since the decibel is a logarithmic unit, then we add or subtract decibels to achieve multiplication or division. We never multiply or divide decibels. A power increase would be represented by a positive dB number. A power decrease would be represented by a negative dB number.

Here is Equation 1 solved for every possible permutation.

$dB = 10 * \log_{10}(P2/P1)$ ex. express a power ratio of 0.5 in dB: (-3)

$P2/P1 = 10^{dB/10}$ ex. what power ratio is represented by 3 dB: (2)

$P1 = P2 / 10^{dB/10}$ ex. P2 is 30 mW and is 6 dB lower than P1: (P1 = 119.4 mW)

$P2 = P1 * 10^{dB/10}$ ex. P1 is 40 mW and is 8 dB higher than P2: (P2 = 6.34 mW) (note: by definition dB is positive if P2 > P1, otherwise it is negative. Another way of stating this problem is that P2 is 8 dB lower than P1 (40 mW). Be careful – it is very easy to be tricked on the sign to use.)

Introduction to Decibels

Homework decibel calculations (with answers)

1. Audio speakers are often rated dBa at 1 meter distance for 1 Watt of power. The typical range is from about 78 to 92 dBa with the higher numbers representing a higher efficiency speaker (not necessarily a higher fidelity speaker). If a certain speaker is rated at 84 dBa, how much power is needed for it to put out a very loud sound level of 110 dBa (assuming the speaker can actually handle this power)? (26 db over 1 watt = 398 watts)

2. If an audio system puts out 6 Watts during a loud passage of music and a soft passage of the music is 50 dB below the loud level, how much power does the amplifier put out during the soft passage (believe it or not, this is quite audible)? (60 uW)

3. If at a certain frequency, a cable has a rated loss of 5 dB per 100 feet, how much power will be delivered to an antenna from a transmitter that puts out 10 watts if the cable run between transmitter and antenna is 40 feet? (6.3 W)

4. If an antenna delivers 10 dBuV to a receiver with input an impedance of 50 Ohms, what is the signal level in dBm? (-97 dBm)

5. What is the voltage level of a 0 dBm signal for 50 Ohms, 75 Ohms, and 600 Ohms? (0.224, 0.274, 0.775 Vrms)

6. Express 1 Watt in dBm. (30 dBm)

7. A person is wearing a set of headphones in a perfectly quiet room. If 100 mW power into the headphones produces a sound level of 90 dBa, what power level would produce a 0 dBa level (the threshold of hearing)? (100 pW)

8. Express 20 Volts in dBV. (26 dBV)

9. A certain instrument has an input impedance of 50 Ohms and is rated for a maximum input of 10 dBm. What is the maximum voltage that can be applied to the instrument? (0.707 Vrms)

10. If the voltage across a certain impedance is increased from 5 to 7 Volts, what is the power increase in dB? (2.9 dB)