

# Circuits for Controlling Slope and Offset

by Kenneth A. Kuhn  
March 13, 2011

## Introduction

In signal processing the most common type of process needed is to adjust the gain (slope of the transfer function) and remove an offset voltage. The math is identical to that of the familiar straight line,  $y = mx + b$ . Equation 1 is an adaptation for voltages.

$$V_o = V_{in} * \text{Gain} + V_{offset} \quad (1)$$

The most general op-amp circuit that can achieve any value of gain and offset is shown in Figure 1. Any specific transfer function like Equation 1 can always be implemented with fewer than the five resistors shown as summarized below.

R1 and R2 Always required

R3 Required if non-inverting gain is to be higher than inverting gain

R4 and R5 Required if non-inverting gain is to be less than unity

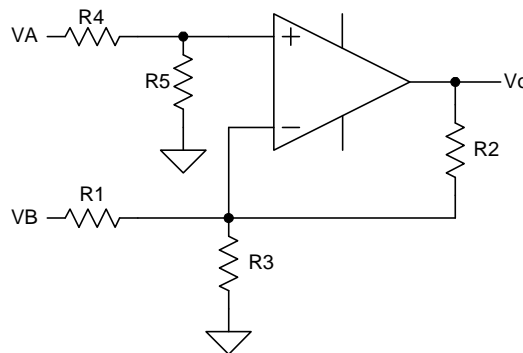


Figure 1: Generic Signal Processing Circuit

## A simple approach when the slope is positive

Figure 2 shows the basic circuit to provide non-inverting gain and remove an output offset. The input voltage is  $V_{in} = 0.003 * T + 0.8$ . The variable, T represents the temperature in Celsius. The 0.003 factor is the input slope and has units of volts per C. There is an offset of 0.8 volts as part of the temperature sensor. The desired output of the op-amp is  $V_o = 0.012 * T$  with no offset. Thus, for a room temperature of 25 C the output voltage would be 0.25 volts.

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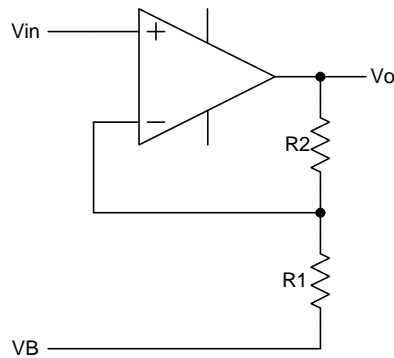


Figure 2: Simple Circuit for non-inverting Gain

The design problem is to determine values for  $R_1$ ,  $R_2$ , and  $V_B$ . The non-inverting gain is the output slope factor divided by the input slope factor or  $0.012/0.003 = 4.00$ . Thus,  $R_2/R_1$  should be 3.00. There are a wide variety of resistors that can achieve this ratio. All we have to do is pick a reasonable one. We choose  $R_2$  to be 30K and then  $R_1$  calculates to be 10K. We could have chosen  $R_1$  first and then calculated  $R_2$  – it really makes no difference.

With a gain of 4.00 then  $V_o = 0.012 \cdot T + 3.2$ . We need to remove the 3.2 volt offset. What value of  $V_B$  will accomplish that? By superposition, the effect of  $V_B$  on the output voltage is  $-V_B \cdot (R_2/R_1)$ . This effect needs to be -3.2 volts to eliminate the 3.2 volt offset. Since  $(R_2/R_1)$  is fixed at 3.00 then  $V_B$  must be  $3.2/3.00 = 1.067$  volts. This completes our design. It is left as an exercise for the student to check this to be sure that it works.

### Practice problems:

1. Given:  $V_{in} = 100 \cdot X - 3.33$ . The output of the amplifier is to be  $V_o = 500 \cdot X$ . Use  $R_2 = 100K$  and find the required values for  $R_1$  and  $V_B$ . Hint: Do not be surprised when  $V_B$  computes to be a negative voltage.
2. The transfer function of the circuit is to be  $25.64 \cdot (V_{in} - 0.1)$ . Use  $R_2 = 100K$  and find the required values for  $R_1$  and  $V_B$ .

Rework the above problems if  $R_1$  was chosen to be 10K in both cases.

### Using a standard voltage for $V_B$ when the slope is positive

In the previous examples,  $V_B$  computed to be some odd voltage. This is an illustration of the fact that the simple and obvious approaches do not necessarily have the best operation. Less obvious, but more advanced circuits do. It is generally desirable for  $V_B$  to be a standard voltage such as 1.250, 2.500, or 5.000 that are available from precision voltage references (the negatives of these voltages are also easily created). How do we modify the circuit for this situation? **Important reasoning:** No matter what we do, the

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non-inverting gain of the circuit must remain the same. If we choose a  $V_B$  higher than required by the previous examples, then the  $R_2/R_1$  ratio must be reduced to achieve the same effect on the output offset voltage. The reduction in non-inverting gain can be corrected by adding  $R_3$  as shown in Figure 3.

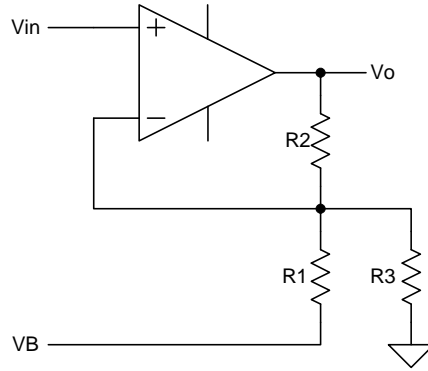


Figure 3: Advanced Circuit for non-inverting Gain

Equation 2 is the transfer function of the circuit shown in Figure 3.

$$V_o = V_{in} * \left(1 + \frac{R_2}{R_1 \parallel R_3}\right) - V_B * \left(\frac{R_2}{R_1}\right) \quad (2)$$

In all cases,  $V_B$  will be chosen (rather than calculated) to be a standard reference voltage. The design problem is that we have three components to meet two criteria. Infinite solutions are possible. However, all we need is a practical solution.

The example of the previous section will be reworked using this new concept. We know that the non-inverting gain is going to be 4.00. But now we first calculate  $(R_2/R_1)$  to satisfy the needed offset. For this example  $R_2/R_1 = (3.2/5.00) = 0.640$ . If we chose  $R_2$  to be 30K as before, then  $R_1$  is  $30K/0.640 = 46.88K$ .

With the value of  $R_1$  and  $R_2$  known then  $R_3$  can be calculated to provide the desired non-inverting gain. We do this in two steps. First, we calculate the value of  $(R_1 \parallel R_3)$  for the desired non-inverting gain as before to be  $30K/3 = 10K$ . Next, we calculate  $R_3$  using the equation for parallel resistance in reverse.

$$R_3 = \frac{46.88K * 10K}{46.88K - 10K} = 12.71K$$

The final step would be to consider the required accuracy of the transfer function and either round the resistors to standard values if low accuracy is acceptable or provide some method to fine adjust the values if high accuracy is required. That step will be the topic of another note.

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### Practice problems:

Rework the previous problems using this method and with  $V_B = 5.00$  volts (positive or negative as required). You will be calculating the value for  $R_3$  instead of  $V_B$ . Then rework them again using  $R_1 = 10K$ .

### A simple approach when the slope is negative

In all of the previous examples the slopes were positive. There are situations where the input slope is negative and a slope inversion is required to make a positive output slope. The circuits and concepts are identical to the previous examples except that  $V_{in}$  and  $V_B$  are applied differently as shown in Figure 4.

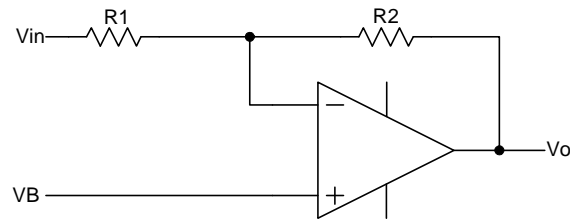


Figure 4: Simple Circuit for inverting Gain

As an example  $V_{in} = 0.8 - 0.0025 * T$  where  $T$  is the temperature in Celsius and the 0.0025 factor has units of volts per C. The desired output voltage is  $0.01 * T$  which means that if the temperature is 40 C then the output voltage would be 0.4 volts.

The design problem is to determine values for  $R_1$ ,  $R_2$ , and  $V_B$ . The inverting gain is the output slope factor divided by the input slope factor or  $0.010 / -0.0025 = -4.00$ . Thus,  $R_2 / R_1$  should be 4.00. There are a wide variety of resistors that can achieve this ratio. All we have to do is pick a reasonable one. We choose  $R_2$  to be 40K and then  $R_1$  calculates to be 10K. We could have chosen  $R_1$  first and then calculated  $R_2$  – it really makes no difference.

With a gain of -4.00 then  $V_o = 0.010 * T - 3.20$ . We need to remove the -3.2 volt offset. What value of  $V_B$  will accomplish that? By superposition, the effect of  $V_B$  on the output voltage is  $V_B * (1 + R_2 / R_1)$ . This effect needs to be +3.2 volts to eliminate the -3.2 volt offset. Since  $(R_2 / R_1)$  is fixed at 4.00 then  $V_B$  must be  $3.2 / 5.00 = 0.640$  volts. This completes our design. It is left as an exercise for the student to check this to be sure that it works.

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### Practice problems:

3. Given:  $V_{in} = -100 \cdot X - 3.33$ . The output of the amplifier is to be  $V_o = 500 \cdot X$ . Use  $R_2 = 100K$  and find the required values for  $R_1$  and  $V_B$ . Hint: Do not be surprised when  $V_B$  computes to be a negative voltage.
4. The transfer function of the circuit is to be  $-25.64 \cdot (V_{in} - 0.1)$ . Use  $R_2 = 100K$  and find the required values for  $R_1$  and  $V_B$ .

Rework the above problems if  $R_1$  was chosen to be 10K in both cases.

### Using a standard voltage for $V_B$ when the slope is negative

In the previous examples,  $V_B$  computed to be some odd voltage. This is an illustration of the fact that the simple and obvious approaches do not necessarily have the best operation. Less obvious, but more advanced circuits do. It is generally desirable for  $V_B$  to be a standard voltage such as 1.250, 2.500, or 5.000 that are available from precision voltage references (the negatives of these voltages are also easily created). How do we modify the circuit for this situation? **Important reasoning:** No matter what we do, the inverting gain of the circuit must remain the same. Keep in mind that  $R_3$  has no effect on the inverting gain. If we choose a  $V_B$  lower than required by the previous examples, then  $R_3$  can be used to increase the non-inverting gain on  $V_B$  to achieve the desired effect on the output offset voltage. However, this approach fails if  $V_B$  is chosen to be a higher magnitude than required in the previous examples. The next section illustrates how to handle that situation.

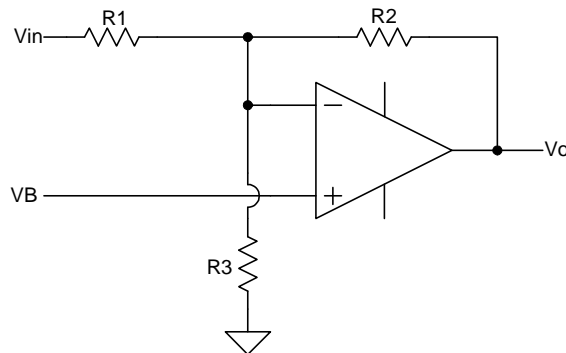


Figure 4: Advanced Circuit for inverting Gain

Equation 3 is the transfer function of the circuit shown in Figure 4.

$$V_o = V_B * \left(1 + \frac{R_2}{R_1 \parallel R_3}\right) - V_{in} * \left(\frac{R_2}{R_1}\right) \quad (3)$$

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In all cases,  $V_B$  will be chosen (rather than calculated) to be a standard reference voltage. The design problem is that we have three components to meet two criteria. Infinite solutions are possible. However, all we need is a practical solution.

The example of the previous section will be reworked using this new concept. We know that the inverting gain is going to be  $-4.00$ . This means that the ratio,  $R_2/R_1$ , remains the same and  $R_1 = 10K$  and  $R_2 = 40K$  as before. The output voltage is  $V_o = 0.010 * T - 3.20$ .

The next question is how to eliminate the  $-3.2$  output offset voltage. There is a  $0.500$  reference voltage available (how that was derived is not important here). By superposition the required non-inverting gain on the  $0.500$  reference voltage is  $3.2/0.500 = 6.40$ . The ratio,  $R_2 / (R_1 \parallel R_3)$  will be  $5.40$ . With the value of  $R_1$  and  $R_2$  known then  $R_3$  can be calculated to provide the desired non-inverting gain. We do this in two steps. First, we calculate the value of  $(R_1 \parallel R_3)$  for the desired non-inverting gain as before to be  $40K/5.4 = 7.407K$ . Next, we calculate  $R_3$  using the equation for parallel resistance in reverse.

$$R_3 = \frac{10K * 7.047K}{10K - 7.047K} = 25.08K$$

The final step would be to consider the required accuracy of the transfer function and either round the resistors to standard values if low accuracy is acceptable or provide some method to fine adjust the values if high accuracy is required. That step will be the topic of another note.

### Practice problems:

Rework the previous problems using this method and with  $V_B = 0.1$  volts (positive or negative as required). You will be calculating the value for  $R_3$  instead of  $V_B$ . Then rework them again using  $R_1 = 10K$ . Do not be surprised if you calculate a negative resistor value for  $R_3$  in some cases – the math is telling you that the circuit can not provide the desired result. A different circuit is needed as described in the next section.

### General case solution for inverting slopes for any $V_B$

For the previous scheme to work for typical situations the magnitude of  $V_B$  generally must be much less than one volt – not practical when using standard voltage references. The solution is to use a different circuit as shown in Figure 5 where the gains of the different paths can be independently made as high or low as necessary.

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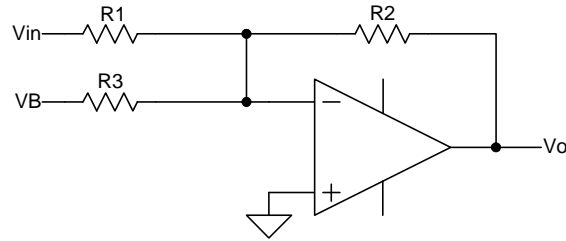


Figure \_: Generic Circuit for Inverting Gain

We will use the previous example and choose  $V_B$  to be -5.00 volts (we need a negative voltage). As before,  $R_2/R_1$  is 4.00 and  $R_1 = 10\text{K}$  and  $R_2 = 40\text{K}$ . The required gain from the -5.00 volt input to make +3.2 volts on the output is  $3.2/(-5.00) = -0.640$ . Thus,  $R_2/R_3$  should equal 0.640 and  $R_3$  then calculates to be 62.5K.

### General problems:

The student will have to choose which of the methods discussed in this note apply to each problem. In some cases more than one method can be used. The important thing is not which method you use but that you understand how to pick an appropriate method and apply it. In some cases you may have to include the  $R_4, R_5$  voltage divider from the generic circuit. Work the problems both ways – without a standard reference voltage ( $R_3$  not needed) and with a standard reference voltage: 1.250, 2.500, or 5.000 and  $R_3$  will be needed. In some cases your chosen value might be too high or too low – you will compute negative resistors. The problems are rigged so that there is a lot of room to make errors in the approach so do not be surprised if a particular approach you take ends up not working. That just means you should try an alternate approach. That is just part of the learning process – mistakes are going to happen – but that is OK – just understand what you did wrong and learn. Answers are not provided. You should be able to set up a spreadsheet with the standard transfer functions so that you can enter your results and prove that you are right (or wrong). In real life no one gives you the answers – it is the job of the engineer to determine the answers and prove it.

1.  $V_{in} = 0.025 * X - 4.2$ .  $V_o = 0.2 * X$ .
2.  $V_{in} = 0.00115 * X + .56$ .  $V_o = 0.01 * X$
3.  $V_{in} = 1.2 - 0.004X$ .  $V_o = 0.25 * X$
4.  $V_{in} = 4 - 0.1X$ .  $V_o = X$
5.  $V_{in} = 2.5 * X - 1$ .  $V_o = X$
6.  $V_{in} = -2.5X + 1$ .  $V_o = X$