Frequency Filters

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A filter in general passes a desired quantity and stops an undesired quantity. For example, a fuel filter in a car passes gasoline and stops dirt and junk. An air filter in either a car or home passes air and stops dust and dirt. A ticket collector at a theater passes those with tickets and stops those without. The filters we will focus on pass signals in a desired frequency band and reject signals in an undesired frequency band. These filters are called frequency filters.

There are five basic frequency filter types:

Low-pass Signals from DC to an upper cutoff frequency are passed and signals at higher frequencies are attenuated.

High-pass The inverse of a low-pass filter. Signals below a specified cutoff frequency are attenuated and signals at higher frequencies are passed. No high-pass filter can have response to infinite frequency so there is an upper low-pass response region in practice that is usually not accounted for in our model.

Band-pass Signals above a lower cutoff frequency and below an upper cutoff frequency are passed and signals at lower or higher frequencies are attenuated. There are two types of band-pass filters: wide-band and narrow-band. Wide-band filters are built using a cascade of a high-pass filter that cuts off at the low frequency and a low-pass filter that cuts off at the high frequency. Narrow-band filters are built using high-Q resonators at or near the center frequency.

Band-reject The inverse of a band-pass filter. Signals between a lower cutoff frequency and an upper cutoff frequency are attenuated and signals at lower or higher frequencies are passed. There are two types of band-reject filters: wide-band and narrow-band. Wide-band filters are built using a parallel network that produces the sum of a high-pass filter that cuts off at the high frequency and a low-pass filter that cuts-off at the low frequency. Narrow-band filters are built using high-Q resonators at or near the center frequency.

All-pass Signals at all frequencies are passed but the filter does have a useful phase response. This filter is useful in correcting phase response of the other filter types and also as a time delay element (delay line). The all-pass filter ultimately turns into a low-pass filter as practical limitations prevent the filter from having response to infinite frequency. The simplest example is a transmission line.

Frequency Filters

The operation of a frequency filter is divided into three regions as illustrated in Figure 1.

Pass-band Signals in this range of frequencies are passed with very little attenuation, often no more than five or ten percent.

Stop-band Signals in this range of frequencies are attenuated by at least a specified amount. Although specifications can vary widely, we generally think of signals in the stop band being attenuated at least 20 dB or perhaps over 40 dB.

Transition-band This is the region between the pass and stop bands and other than some sort of transition from pass-band to stop-band attenuation, there is no control of the magnitude response. The response type chosen and the filter order will determine the width of the transition-band.

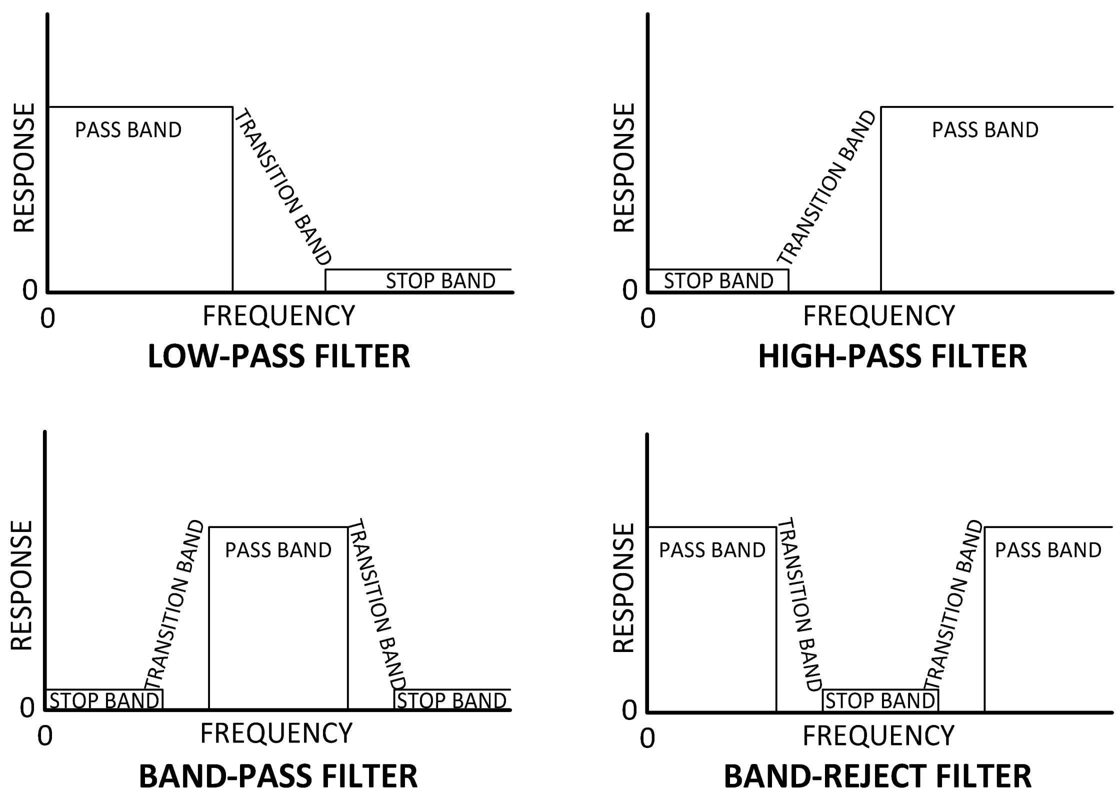

Figure 1: Filter Types

The **cutoff frequency** is normally defined as the frequency where the normalized filter pass-band response has been reduced by a factor of 0.707 (1/sqrt(2)) or -3 dB. This is also known as the half-power frequency because the signal power has been reduced by a factor of 0.5 (0.707^2). There are a few necessary exceptions. An all-pass filter does just what its name implies but does have a phase cutoff frequency. In a Chebyshev response type filter the cutoff frequency is defined as the frequency where the response falls below the specified ripple.

Frequency Filters

Filter Order

The order of a filter determines how steep the response falls off in the stop-band. The higher the order, the steeper the slope of the amplitude (or phase) response versus frequency. The order of a filter is the highest power of s that occurs in the transfer function polynomial denominator. Sometimes a filter needs to be of order 20 or more to achieve a particular specification.

Order	Slope
1	-6 dB per octave or -20 dB per decade
2	-12 dB per octave or -40 dB per decade
3	-18 dB per octave or -60 dB per decade
4	-24 dB per octave or -80 dB per decade
5	-30 dB per octave or -100 dB per decade
6	-36 dB per octave or -120 dB per decade
7	-42 dB per octave or -140 dB per decade
8	-48 dB per octave or -160 dB per decade
9	-54 dB per octave or -180 dB per decade

Response Types

Each type of filter can be implemented in different ways. Each implementation has its own advantages and disadvantages. No implementation has all the good qualities one may want so compromise is necessary. Response types with narrow transition bands in the frequency domain are very frequency selective and have the advantage of requiring fewer components to achieve a given specification but have undesirable overshoot and ringing in their time domain responses because the poles are located close to the jw axis. Response types with wide transition bands (poles well away from the jw axis) have little or no overshoot in their time domain responses but require the maximum number of components to achieve a given specification in the frequency domain and may not be able to achieve some specifications at all. A concept called “brick wall filter” appears often in filter literature. This is a theoretical concept of a filter that has infinite order and zero transition band width. The filter shape is rectangular resembling the diagrams in Figure 1 except that the stop band amplitude would be zero and there would be no transition band. The following discusses some of the most common filter response types.

Butterworth Response

The Butterworth response is known as the maximally flat response. The response has a smooth roll-off with no ripple. The transition-band is medium in width. The cutoff of the filter is not sharp and the phase response is not too bad for low order filters. This is the easiest filter type to compute pole or zero locations as they fall exactly on a circle whose radius in the s-plane is the cutoff frequency. The phase response is fair and there is moderate overshoot and ringing to a step function. This filter is a good compromise when the time domain response is of medium importance relative to the frequency domain response. A British engineer, Stephen Butterworth, first used this filter in 1930.

An example of a sixth-order Butterworth low-pass filter is shown in Figure 2. This filter is comprised of a cascade (i.e. product) of three second-order sections – the response of each is shown in the thin lines. The bold line is the composite response. Note that the three individual second-order responses range from underdamped to overdamped with a natural frequency of 10 kHz – the cutoff frequency of the filter. The second-order responses fall off at 40 dB per decade while the composite response falls off at 120 dB per decade.

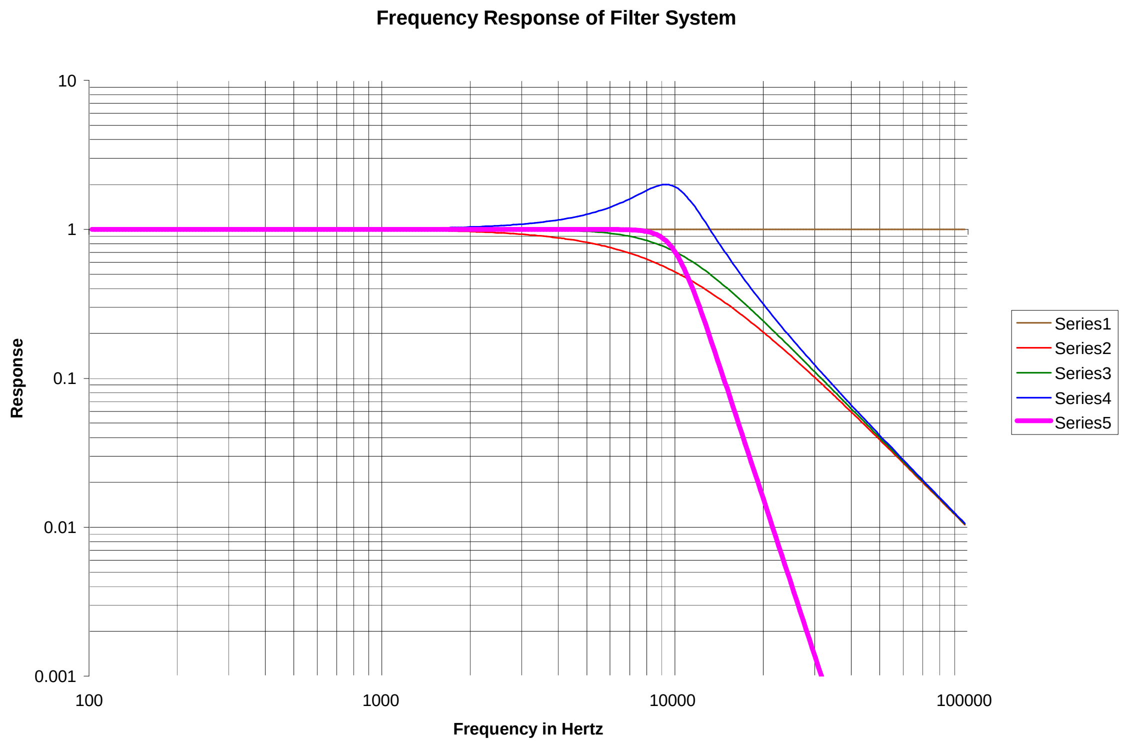

Figure 2: Sixth-order Butterworth Low-pass Filter with 10 kHz cutoff

Chebyshev Response

The Chebyshev response is based on a Chebyshev polynomial that has equal ripple about the desired pass-band response. This concept produces a more accurate approximation to the desired pass-band response. The transition-band is much narrower than that of the Butterworth and the required filter order is generally less than that of a Butterworth type for a given specification – thus saving parts. The cutoff frequency of a Chebyshev filter is defined as the frequency that the magnitude response drops below the specified pass-band ripple. Pass-band ripple is commonly specified between about 0.01 dB and 1 dB. The phase response is poor and there is significant overshoot and ringing to a step function. This filter is useful only when the time domain response is not important. A

Frequency Filters

Russian mathematician, P. L. Chebyshev, first used Chebyshev polynomials in 1899 while studying the construction of steam engines. Chebyshev polynomials have many other uses besides filter approximations. An example of a sixth-order Chebyshev low-pass filter with 1 dB of ripple is shown in Figure 3. Note the very sharp cutoff characteristic. Note also that the three second-order sections have different cutoff frequencies and have lower damping compared to the Butterworth. The low damping is what makes the filter ring so much to a step signal.

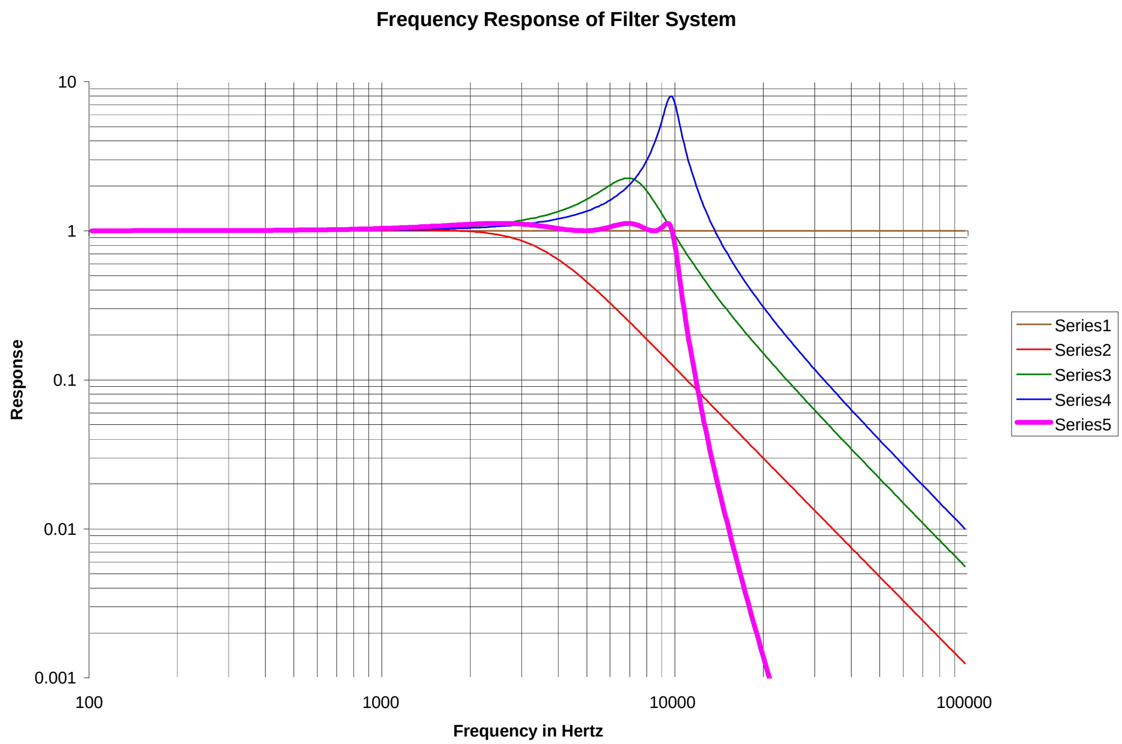

Figure 3: Sixth-order Chebyshev 1 dB Low-pass Filter with 10 kHz cutoff

Elliptical Response

The elliptical response has an extremely narrow transition-band and is very useful when the filter must be very frequency selective. There are transmission zeros in the stop-band and these produce the very high initial cutoff response – thus the narrow transition band. There is ripple in the stop-band response due to the zeros. The key feature of the elliptical response is the very narrow transition-band. Thus, an elliptical response type filter requires the lowest order and therefore the smallest number of components of any of the response types. The price paid is a very poor time domain response with significant overshoot and ringing to a step signal. An elliptical filter would be a very poor choice for pulse signals.

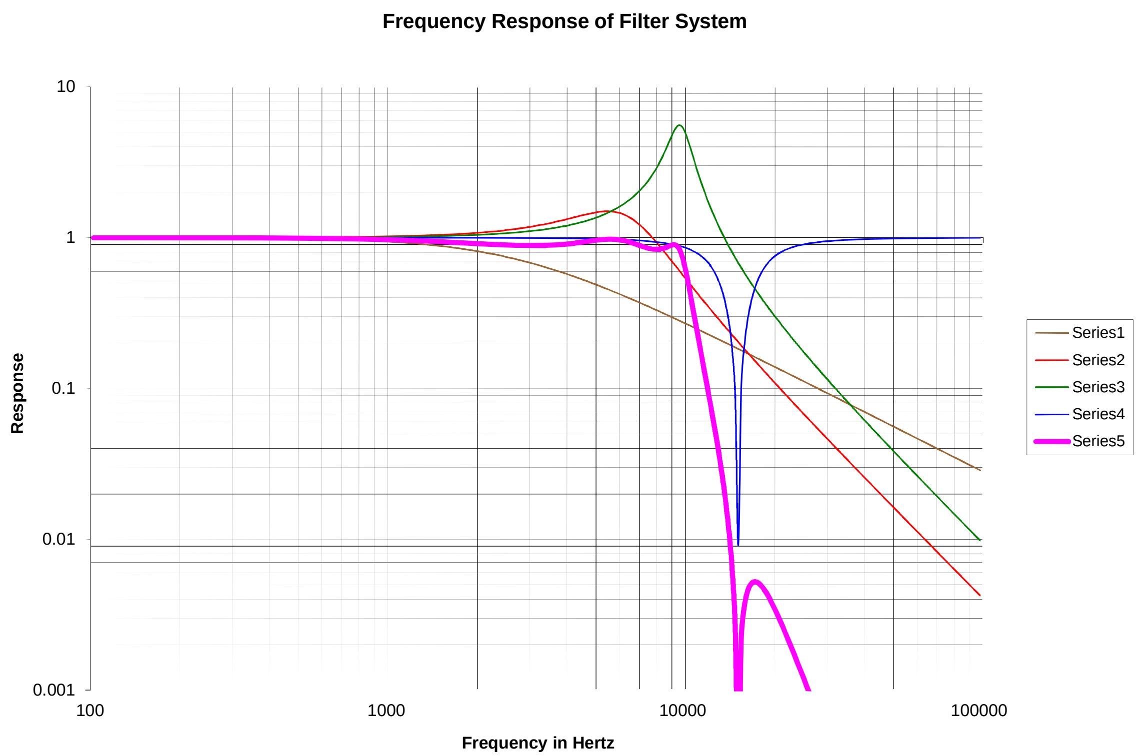

Figure 4: Example of the narrow transition band created by a zero in the stop band

Bessel Response (closely related to Gaussian Response)

The Bessel response emphasizes linear phase response (constant time delay) at the expense of magnitude response (i.e. sharpness of cutoff). The linear phase characteristic causes the time-domain response to have little or no overshoot to a step function thus making this filter very attractive for filtering pulse waveforms. This filter type has a very wide transition-band (even when the order is high) and is very poor at frequency selectivity. A hybrid filter constructed by interpolating the poles of a Bessel and Butterworth filter often offers the best compromise between time and frequency response when both responses are important. An example of a sixth-order Bessel low-pass filter is shown in Figure 5. Note that the three second-order sections have high damping. The high damping is what gives the filter such an excellent time domain response to step signals.

The Bessel response can be thought of as a finite approximation and with some minor tweaks to a theoretical Gaussian response. A true Gaussian response requires an infinite number of poles but it is practical to truncate after a reasonable number. Both types are used and the differences between them are generally subtle.

The Bessel/Gaussian response has a useful characteristic known as linear phase. Linear phase means that the phase lag of the filter increases at a constant rate with frequency. With linear phase the time delay of low and high frequencies through the filter is the same. Constant delay with frequency preserves the wave shape of signals within the pass-band. That is very important for pulse type signals. Without linear phase, high frequency details in analog video would be displaced from low frequency details resulting in a smeared image. The other filter types do not have linear phase and in general non-linear phase performance worsens as the transition band is narrowed.

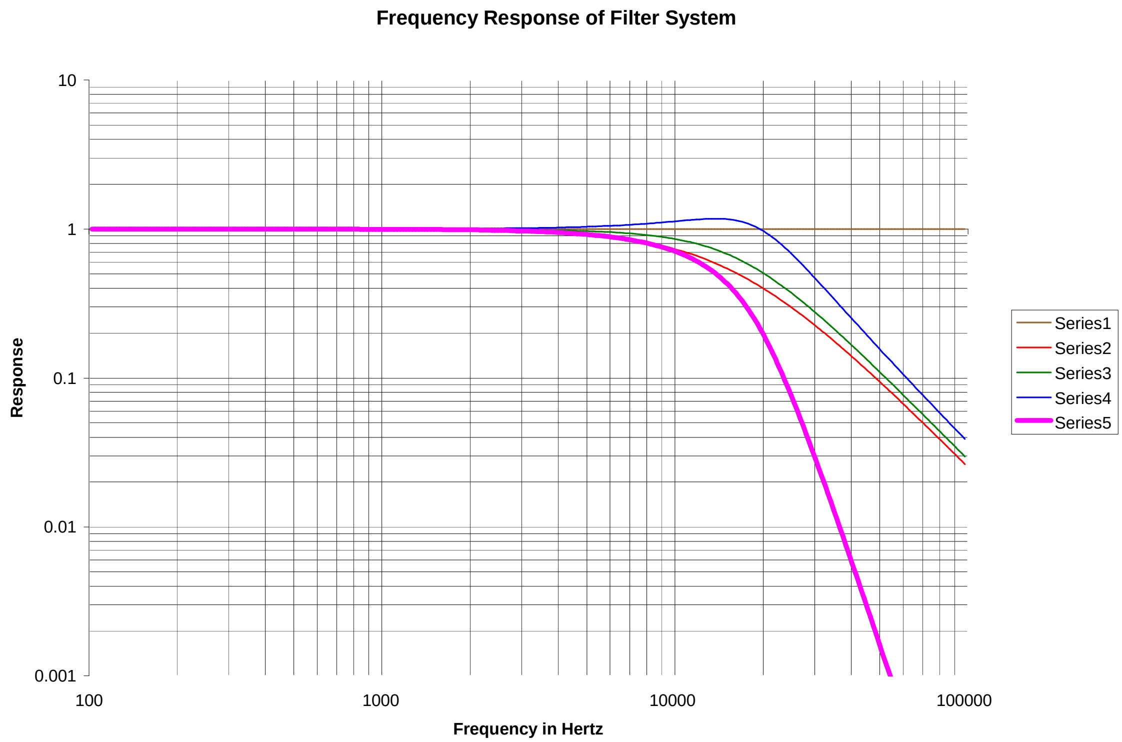

Figure 5: Sixth-order Bessel Low-pass Filter with 10 kHz cutoff

Other Response Types

Besides the popular response types listed here, there are many possible response types that can be created by interpolation between these or other methods to achieve particular tradeoffs between the frequency and time domain responses. For example, a compromise filter might begin as a Bessel response for good time domain characteristics but transition to a Butterworth or even Chebyshev response into the stop band for steeper response.

Filter Polynomials

Frequency filters are constructed using a cascade of second-order transfer functions (and a first-order function if the filter order is and odd number). It should be noted that low-pass, high-pass, wide-band band-pass or wide-band band-reject, and all-pass filters can be either even or odd order but narrow-band band-pass and band-reject filters can only be even order since each resonator section is second-order. The following polynomials illustrate the basic forms:

Frequency Filters

First order forms:

Low-pass: $\dfrac{a}{s + a}$

High-pass: $\dfrac{s}{s + a}$

All-pass: $\dfrac{s - a}{s + a}$

Second order forms:

Low-pass: $\dfrac{w_n^2}{s^2 + 2\xi w_n s + w_n^2}$

High-pass: $\dfrac{s^2}{s^2 + 2\xi w_n s + w_n^2}$

Band-pass: $\dfrac{2\xi w_n s}{s^2 + 2\xi w_n s + w_n^2}$ (narrow-band type)

In many implementations, there is also a scalar proportional to Q^2
$Q = 1 / (2\xi)$

Band-reject: $\dfrac{s^2 + w_n^2}{s^2 + 2\xi w_n s + w_n^2}$ (narrow-band type)

All-pass: $\dfrac{s^2 - 2\xi w_n s + w_n^2}{s^2 + 2\xi w_n s + w_n^2}$

Instead of ξ, some people prefer to work with Q instead. $Q = 1 / 2\xi$ or $\xi = 1 / 2Q$ or $2\xi = 1 / Q$

Pole locations

The pole locations for three common low-pass filters are shown in Figure 6. Note that the poles for a Bessel or Gaussian filter are far from the jw axis – thus the very low ringing in the step response. Note that the poles for the Butterworth filter lie on a perfect circle. Note that the poles for the Chebyshev filter are very close to the jw axis – thus the high ringing in the step response.

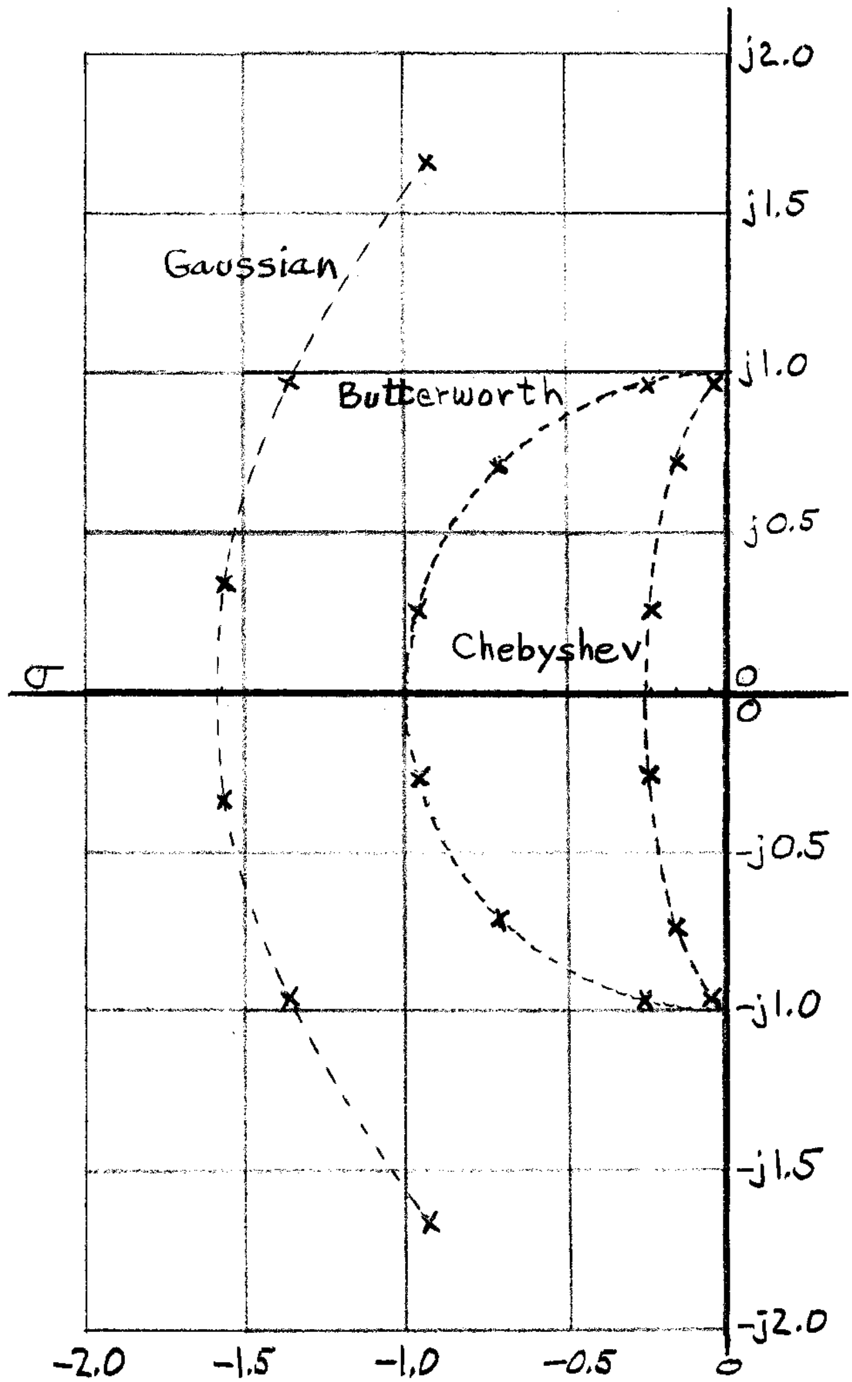

Figure 6: Pole Locations for Several Sixth-order Low-pass Filters

Deriving High-pass, Band-pass, and Band-reject Filters

The starting point for all filter design is low-pass. The filter specifications are transformed to an equivalent low-pass form and the poles determined. Then a transformation is applied to the low-pass pole locations to produce the required poles and zeros for the desired filter type. These transformations are not difficult but are beyond the scope of this student article.

Time Domain Response of Filters

As previously discussed the various filter implementations have different time domain responses as illustrated in the following figures.

Figure 7: Time domain response of 4th order 1 kHz Chebyshev 1 dB low-pass filter

Figure 8: Time domain response of 4th order 1 kHz Butterworth low-pass filter

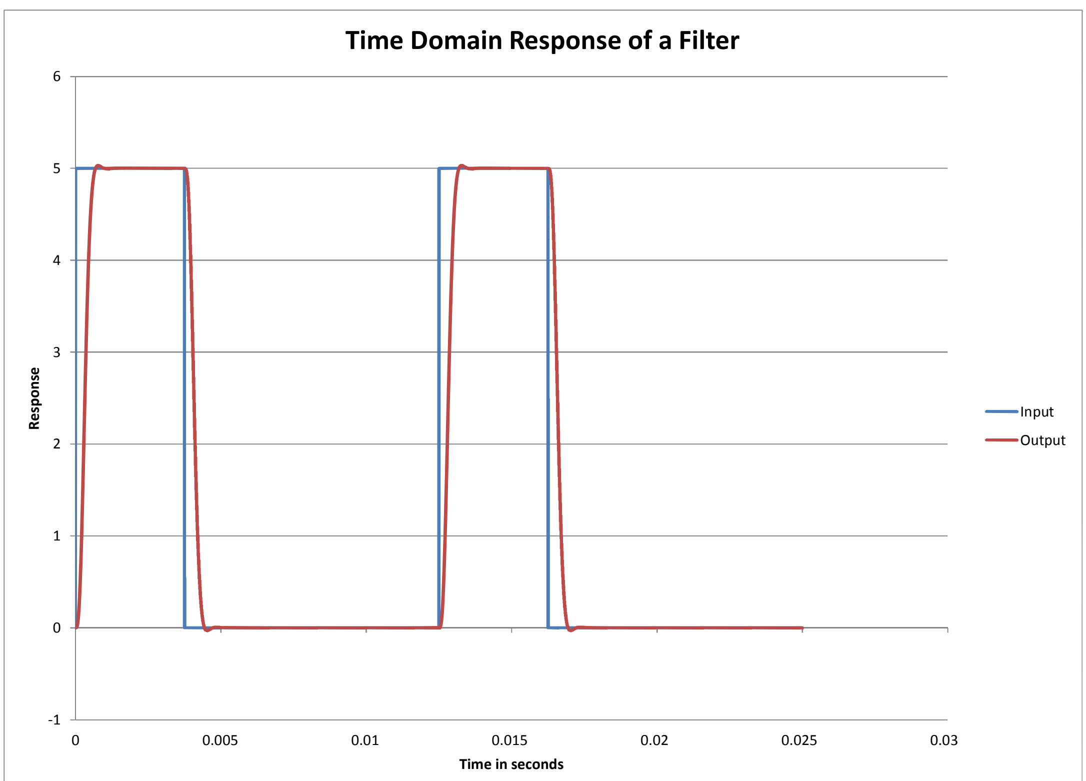

Figure 9: Time domain response of 4th order 1 kHz Bessel or Gaussian low-pass filter

Frequency Filters

Filter circuit implementation

Frequency filters can be built using either passive (i.e. inductors, capacitors, and resistors) or active (i.e. amplifiers, resistors, and capacitors) electronics. It is more practical to build high-frequency filters using passive components and low-frequency filters using active electronics. In this case the dividing line between low and high frequency is several hundred kHz.

Switched capacitor techniques can be used for frequencies less than about 50 kHz. This method is a way to digitally create precision ratios necessary to build a fifth or higher order filter. Usually, all the components are in a single integrated circuit and all the user has to do is supply power and a specific clock frequency related to the desired cutoff frequency. All the design has already been done by the IC manufacturer. It should be noted that switched capacitor filters have some issues which may be problematic. The ultimate attenuation may only be in the 60 to 80 dB range due to filter leakage. There may be artifacts related to the clock in the output. An analog low-pass filter may be needed in front of the switched capacitor filter to reduce aliasing of high-frequency signals in the stop-band. The signal to noise ratio of switched capacitor filters is generally inferior to other filter types. But, when these issues are of minimal concern, switched capacitor filters are a very simple implementation.

Filter design

Filter design consists of:

1. Specifying frequency range of the desired pass-band
2. Specifying the tolerable amplitude variation in the pass-band (typically 10 percent or less)
3. Specifying the frequency range of the desired stop-band
4. Specifying the maximum allowable amplitude response at any frequency in the stop-band
5. Specifying the desired response type: Butterworth, Chebyshev, etc.
6. Calculating the required filter order to achieve the given specifications
7. Determining the pole and zero locations of the filter
8. Determining the component values to implement the filter

Determining filter pole or zero locations using tables

Although the pole or zero locations for a Butterworth or Chebyshev filter are readily calculated, the other filter types require computer iteration. Since the results of the calculations for either a Butterworth or Chebyshev filter are always the same, the results along with the results of computer iteration for the other types are often tabulated in tables. The tables save a considerable amount of computational work and also reduce errors. See a separate article, *Use of Tables for Filter Implementation.*

Frequency Filters

Active Filter Cascading Sequence

High order filters are built by cascading second-order filter sections with possibly a single first-order section if the overall order is odd. Note that each second-order filter section is different. Cascading is the same as multiplying and the order of the sections does not matter mathematically ***but does matter in a very practical sense***. There are two opposite optimums.

In terms of large signal handling the optimum cascade sequence is to begin with the poles farthest to the left of the jw axis and progress towards the axis. This sequence maximizes the dynamic range and prevents inadvertent clipping on large amplitude signals that should make it through the chain unclipped. The thing to keep in mind is that the poles closest to the jw axis cause response peaks (i.e. gain greater than 1). It is important for large amplitude signals near the filter cutoff frequency to be attenuated by the pole pairs further away from the jw axis so that the net effect of the response peaks does not lead to the amplifier going into clipping.

In terms of lowest noise form the individual amplifiers comprising the filter the optimum cascade sequence is to being with the poles closest to the left of the jw axis and progress away from the axis. The low damped poles cause the noise gain of the amplifier to be relatively high. By locating the low damped poles first then the higher damped poles of subsequent stages provides filtering thus lowering the noise from the amplifiers. The pole sequence makes no difference concerning existing noise entering the filter along with a desired signal.

The pole sequence to use depends on which of the above choices is more important for a particular application. There is no universal answer.

Problems that can occur in Filters

The most common filter problem is called, leak-through, and literally means that there is a path from input to output that bypasses the filter components. The result of this is that attenuation in the stop band is not as good as predicted. The primary causes are poor component layout and ground loops. It is one thing to design a good filter. It is usually much more difficult to actually build it.

Another problem that can occur is signal distortion due to non-linear components (either passive or active) in the filter. Even if the non-linearity is in the 0.1 percent range, intermodulation of different signals within the pass-band may be more than is tolerable. The only solution is to design to not have this problem. This generally means avoiding inductors with iron cores and using premium quality operational amplifiers with gain bandwidth products over one hundred times the highest frequency of interest in active filters.

Frequency Filters

Due to poor construction, filters can pickup unwanted signals.   Use of cheap grades of operational amplifiers may result in random noise being added to the desired signal.

See the following web links for more information:

http://en.wikipedia.org/wiki/Butterworth_filter

http://en.wikipedia.org/wiki/Chebyshev_filter

http://en.wikipedia.org/wiki/Elliptic_filter

http://en.wikipedia.org/wiki/Bessel_filter

http://en.wikipedia.org/wiki/Gaussian_filter