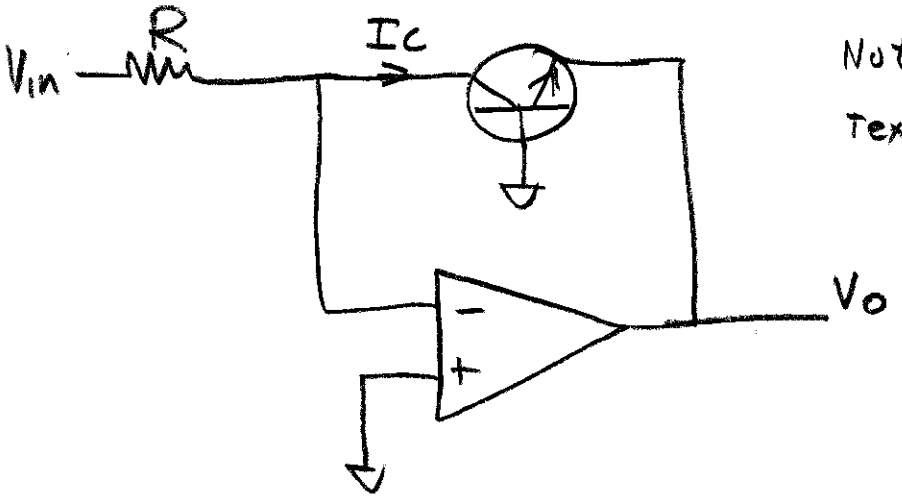
LOGARITHMIC CIRCUIT



Note: NPN CIRCUIT
Text is for PNP

FROM EBERS-MOLL EQUATIONS:

$$I_C = C_{21}\left(e^{V_{EB}/(kT/q)} - 1\right) + C_{22}\left(e^{V_{CB}/(kT/q)} - 1\right)$$

T IS TEMPERATURE IN DEGREES KELVIN

q IS ELECTRON CHARGE = 1.6×10^{-19} COULOMBS

k IS BOLTSMANN'S CONSTANT = 1.38×10^{-23} JOULES/°K

THUS kT/q IS IN UNITS OF JOULES/COULOMB OR VOLTS

NOTE: $V_{EB} = -V_o$.

$V_{CB} = 0$ SO ONLY FIRST TERM OF I_C EQN IS USED

THUS: $I_C = C_{21}\left(e^{-V_o/kT/q} - 1\right)$ WHICH CAN BE WRITTEN AS:

$$I_C + C_{21} = C_{21}\left(e^{-V_o/kT/q}\right)$$

SINCE C_{21} IS TYPICALLY IN THE FEMTO-AMPERE RANGE (10^{-15}) AND I_C IS TYPICALLY BETWEEN 1μA AND 10mA, WE CAN WRITE:

$$I_C = C_{21}\left(e^{-V_o/kT/q}\right)$$

NOTE ALSO THAT $I_C = V_{in}/R$. SO;

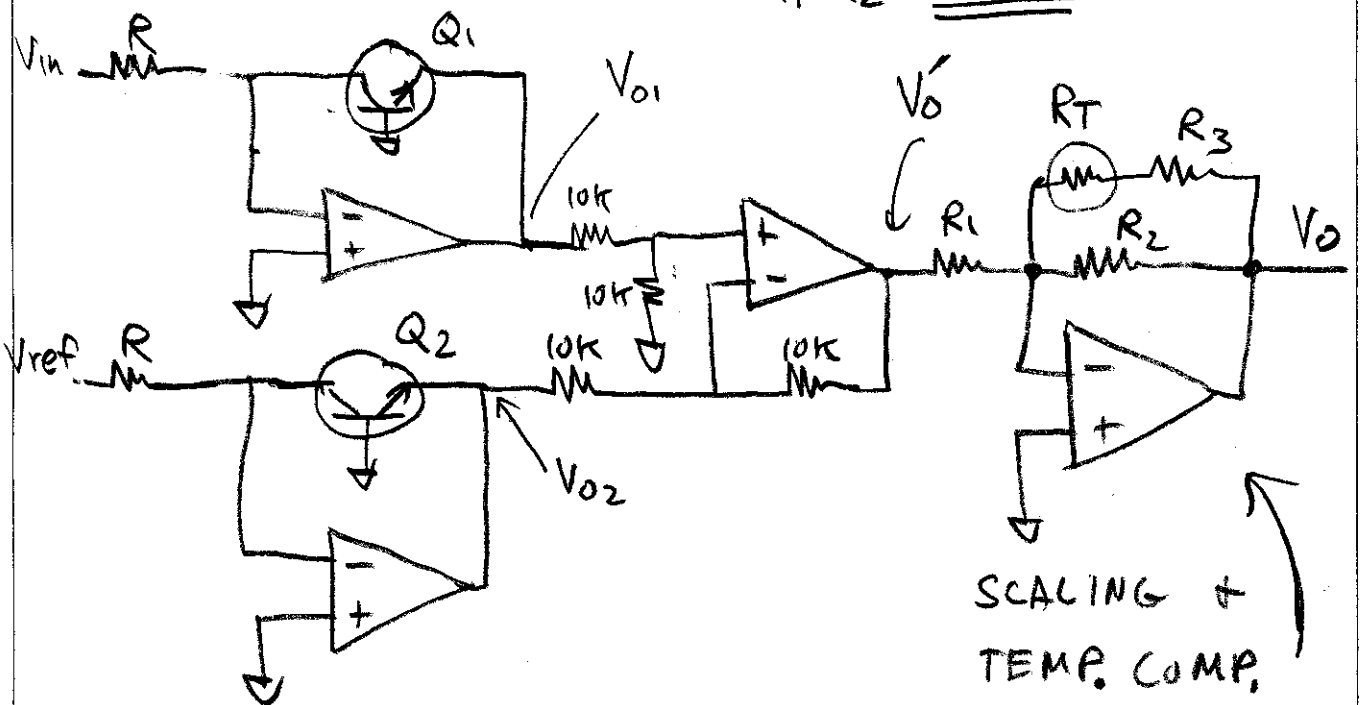
$$C_{21} e^{-V_O/kT/q} = \frac{V_{in}}{R}$$

DIVIDING BY C_{21} AND TAKING LN:

$$\frac{-V_O}{kT/q} = \ln\left(\frac{V_{in}}{C_{21} R}\right) = \ln \frac{V_{in}}{R} - \ln C_{21}$$

THUS: $$\boxed{V_O = -\frac{kT}{q}\left[\ln \frac{V_{in}}{R} - \ln C_{21}\right]}$$

IMPROVED CIRCUIT Q_1, Q_2 matched



$$V_{O1} = -\frac{kT}{q}\left[\ln\left(\frac{V_{in}}{R}\right) - \ln C_{21}\right]$$

$$V_{O2} = -\frac{kT}{q}\left[\ln\left(\frac{V_{ref}}{R}\right) - \ln C_{21}\right]$$

$$V_o' = -\frac{kT}{q}\left[\ln\left(\frac{V_{in}}{R}\right) - \ln C_{21} - \ln\left(\frac{V_{ref}}{R}\right) + \ln C_{21}\right]$$

$$V_o' = -\frac{kT}{q}\left[\ln\left(\frac{V_{in}}{R}\right) - \ln\left(\frac{V_{ref}}{R}\right)\right]$$

$$\boxed{V_o' = -\frac{kT}{q}\left[\ln\left(\frac{V_{in}}{V_{ref}}\right)\right]}$$

NOTE:

1. C_{21} TERM CANCELS IF Q_1, Q_2 MATCHED
2. NOTE SENSITIVITY TO TEMPERATURE

FINAL IMPROVEMENT:

1. ADD SCALING TO PRODUCE DESIRED LOG FUNCTION.
2. USE R_T (THERMISTOR — RESISTANCE DECREASES WITH INCREASE IN TEMP.) SUCH THAT SCALING GAIN DECREASES AT THE RIGHT RATE WITH TEMPERATURE INCREASE TO GREATLY REDUCE SENSITIVITY TO TEMPERATURE.