Noise in Operational Amplifiers

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Introduction

This note describes how to calculate the total output noise voltage in an op-amp circuit. There are three components of the output noise voltage:

- the thermal noise in the resistors (3 terms)
- the op-amp input voltage noise (1 term)
- the op-amp input current noise (2 terms)

The six total terms are calculated individually (using superposition) and summed statistically (instead of linearly summed) to produce the net output noise voltage. Refer to Figure 1 for the circuit being analyzed. Any signal sources connected to R_I or R_S are eliminated by superposition since we are only interested in the output noise voltage.

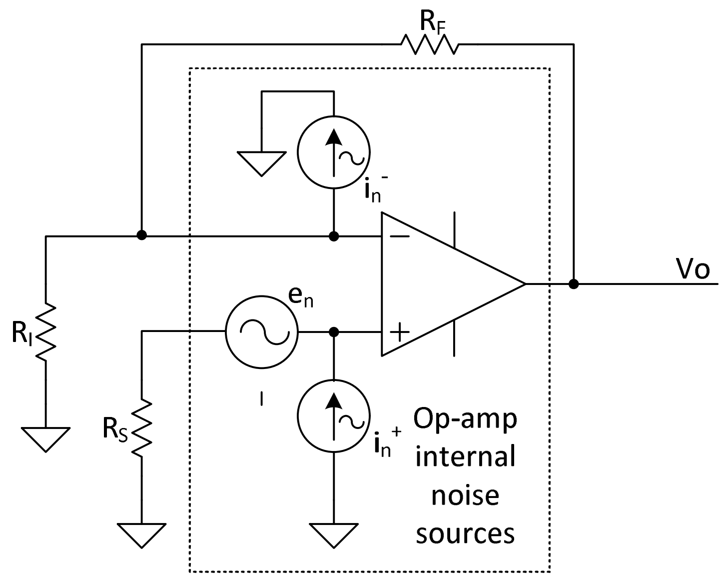

Figure 1: Op-amp noise model

In the following:

k = Boltzmann's constant, 1.38E-23 joules/kelvin
T = Temperature in kelvins
B = Effective noise bandwidth in Hz
R = Resistance in ohms
e_n= op-amp noise voltage in volts per root Hertz
i_n = op-amp current noise in amperes per root Hz

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R_S noise calculation

The circuit is a standard non-inverting amplifier with a gain of (1 + R_F/R_I). The output noise voltage is the noise voltage across R_S multiplied by the non-inverting gain. In some cases, R_S is either zero or is shunted by a bypass capacitor which renders R_S for noise calculations to be practically zero.

$$Vo_n|R_S = \sqrt{4kTBR_S} \times \left(1 + \frac{R_F}{R_I}\right) \tag{1}$$

R_I noise calculation

The circuit is a standard inverting amplifier with a gain of (R_F/R_I). We ignore the negative sign on the gain since we are only interested in magnitude (the phase of noise is generally unimportant). The output noise voltage is the noise voltage noise across R_I multiplied by the inverting gain.

$$Vo_n|R_I = \sqrt{4kTBR_I} \times \left(\frac{R_F}{R_I}\right) \tag{2}$$

In some circuits such as electrometers, R_I is essentially infinity or at least extremely large. An examination of Equation 1 indicates that the output noise voltage for R_I in this case goes to zero. R_I is never taken as zero if it is not present – that is a common error – it is infinity instead.

R_F noise calculation

R_F connects between the output voltage and the virtual ground at Vin- (always 0 volts). Thus, the output noise voltage is the noise voltage across R_F. This term is often negligibly small in ordinary amplifier calculations but can become significant in electrometer circuits where R_F may be very large – in the hundreds to thousands of megohms.

$$Vo_n|R_F = \sqrt{4kTBR_F} \tag{3}$$

Op-amp input noise voltage calculation

All op-amps have an input noise voltage, e_n, provided on the data sheet as a voltage per root Hz – the noise voltage spectral density. To obtain the rms noise voltage we must multiply the data sheet value by the square-root of the noise bandwidth we are using. The output noise voltage is this voltage multiplied by the non-inverting gain.

$$Vo_n|e_n = e_n \times \sqrt{B} \times \left(1 + \frac{R_F}{R_I}\right) \tag{4}$$

Op-amp input current noise calculation

All op-amps have an input noise current, i_n, provided on the data sheet as a amperes per root Hz – the noise current spectral density. To obtain the rms noise current we must multiply the data sheet value by the square-root of the noise bandwidth we are using. Although the magnitude of the noise current is taken as identical for both inputs, the two input noise currents are uncorrelated. Thus, we calculate the output voltage noise caused by each and statistically sum the result.

For the inverting input current noise, the input noise voltage is the input current noise, i_n^-, multiplied by the parallel resistance of R_F and R_I. The voltage at the inverting input is zero since that is a virtual ground. Thus, the effective input noise voltage appears at the non-inverting input and is multiplied by the non-inverting gain. This takes a little thought to understand but the math is straightforward.

$$Vo_n|i_{n^-} = i_n \times \sqrt{B} \times (R_F||R_I) \times \left(1 + \frac{R_F}{R_I}\right) \tag{5a}$$

which simplifies to

$$Vo_n|i_{n^-} = i_n \times \sqrt{B} \times R_F \tag{5b}$$

For the non-inverting input current noise, the input noise voltage is the input current noise, in, multiplied by R_S. The output noise voltage is this input noise voltage multiplied by the non-inverting gain.

$$Vo_n|i_{n^+} = i_n \times \sqrt{B} \times R_S \times \left(1 + \frac{R_F}{R_I}\right) \tag{6}$$

Statistical summation

Since each output noise voltage is uncorrelated to the others, we use statistical summation to combine the results into a single noise voltage. Statistical summation is taking the square root of the sum of each of the individual terms squared as shown in Equation 7.

$$Vo_n = \sqrt{(Vo_n|R_S)^2 + (Vo_n|R_I)^2 + (Vo_n|R_F)^2 + (Vo_n|e_n)^2 + (Vo_n|i_{n^-})^2 + (Vo_n|i_{n^+})^2} \quad (7)$$

The different terms in Equation 7 will have different degrees of significance depending on the noise specifications for the op-amp and the particular resistor values. Any term (prior to squaring) that is more than about three times the magnitude of the next higher term will dominate the result.

Example 1

Determine the net output voltage noise over a 20 kHz noise bandwidth for the following circuit at a temperature of 300 K (27 C or 81 F). R_S = 5 K, R_I = 1 K, R_F = 100K. The op-amp has a voltage noise of 25 nV per root Hz and a current noise of 4 pA per root Hz.

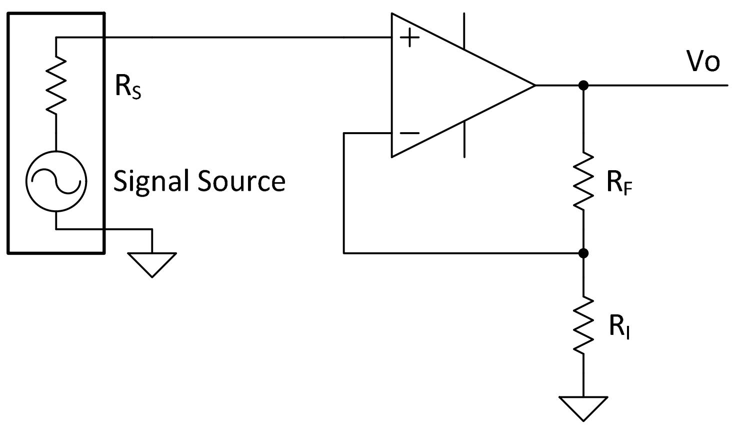

Figure 2: Example problem

Solution:

The inverting gain magnitude is 100
The non-inverting gain magnitude is 101
$Vo_n|R_S$ = 130 uVrms
$Vo_n|R_I$ = 57.6 uVrms
$Vo_n|R_F$ = 5.8 uVrms
$Vo_n|e_n$ = 357 uVrms
$Vo_n|i_n^-$ = 56.6 uVrms
$Vo_n|i_n^+$ = 286 uVrms

Vo_n = 482 uVrms

To prevent a huge error in performing the statistical summation it is better to represent all six intermediate terms in the same units – in this case uVrms.

Example 2

We will rework Example 1 from the perspective of obtaining the answer in terms of the noise voltage spectral density. We do the identical calculations as before but the bandwidth is 1 Hz for all cases. This type of solution is done when the final noise bandwidth is either not yet known or may be changed or as discussed below saves manual computational time.

$Vo_n|R_S$ = 919 nVrms per root Hz
$Vo_n|R_I$ = 407 nVrms per root Hz
$Vo_n|R_F$ = 40.7 nVrms per root Hz
$Vo_n|e_n$ = 400 nVrms per root Hz
$Vo_n|i_n^-$ = 2,000 nVrms per root Hz
$Vo_n|i_n^+$ = 2,500 nVrms per root Hz

Vo_n = 3.42 uVrms per root Hz

To prevent a huge error in performing the statistical summation it is better to represent all six intermediate terms in the same units – in this case nVrms per root Hz.

If the noise bandwidth was 20 kHz then we would multiply the above result by $\sqrt{20{,}000}$ and obtain 482 uVrms – the same answer as before. If the noise bandwidth was changed to be 5,000 Hz then we would multiply the above result by $\sqrt{5{,}000}$ and obtain 242 uVrms without having to recalculate all six intermediate terms.

This example suggests that manual work could be saved by always computing all six intermediate terms in terms of a noise bandwidth of 1 Hz and then doing a single bandwidth adjustment after the total noise voltage density has been determined.