Op-Amp Frequency Response

by Kenneth A. Kuhn
Dec. 30, 2007, rev. Feb. 28, 2009

Introduction

This article explores the effects of the finite frequency response of an op-amp. There is a related spreadsheet written by the author, op_amp_frequency_response.xls, that the student can use to perform a variety of simulations to better understand the complicated nature of this material. The plots shown in this article were created with that spreadsheet. It may take a little care in interpreting the color plots on black and white printouts but all should become clear from the context. The spreadsheet enables up to a third order model which is adequate for many realistic experiments.

Zero order model of an op-amp

The simplest model for an operational amplifier is shown in Equation 1.

$$Vo = (Vin+ - Vin-) * Av \qquad \text{Eq. 1}$$

where:
Vo is the output voltage
Vin+ is the voltage at the non-inverting input
Vin- is the voltage at the inverting input
Av is the DC open loop gain

This model does not include the effect of finite bandwidth of the op-amp.

First order model of an op-amp

This zero order model works fine at low frequencies but is incapable of predicting the performance of an op-amp circuit over frequency. The next step in a model for an operational amplifier includes a first order pole (this pole is always the most dominant) that causes the gain of the amplifier to drop with frequency. This is illustrated in Equation 2 below.

$$Vo = (Vin+ - Vin-) * Av * \frac{1}{T1*s + 1} \qquad \text{Eq. 2}$$

where:
T1 is a time constant in seconds
s is jw

Op-Amp Frequency Response

Equation 2 is a considerable improvement and provides excellent results up to frequencies roughly one-tenth of the gain-bandwidth product of the op-amp. By definition the gain-bandwidth product (GBW) is the product of the bandwidth of the amplifier (-3 dB frequency) and the DC gain of the amplifier (at DC). It has units of Hz and is the theoretical or extrapolated frequency at which the gain of the amplifier has dropped to unity. It is sometimes referred to as the unity gain frequency (FUG). Thus, GBW and FUG can be used interchangeably. It is based on a first order model such that the gain above the cut-off frequency is inversely proportional to frequency. Thus, the gain at a frequency ten times higher is only one-tenth as much. In actuality there is a cascade of poles such that the actual frequency at which the amplifier has a gain of unity is always less than the extrapolated unity gain frequency or GBW. The extrapolated unity gain frequency is used most often in the data sheets as it is easy to work with. Data sheets for wide-band op-amps usually provide some rough data for determining frequency breakpoints of the next most dominate pole.

The open loop gain, Av, of a particular op-amp varies considerably from device to device and also with operating conditions such as temperature, power supply voltage, and other effects. However, the gain-bandwidth product remains fairly constant from device to device but does vary a bit with temperature and power-supply voltage and other effects. Using the first order model and working backwards from GBW we conclude from the definition that the cut-off frequency of the amplifier must be GBW/Av. This turns out to be a very low frequency for most any op-amp. As an example, an op-amp with a GBW of 1 MHz and an Av of 100,000 has a cut-off frequency of 10 Hz. This is illustrated in Figure 1 which shows this amplifier used with a closed-loop non-inverting gain of 100.

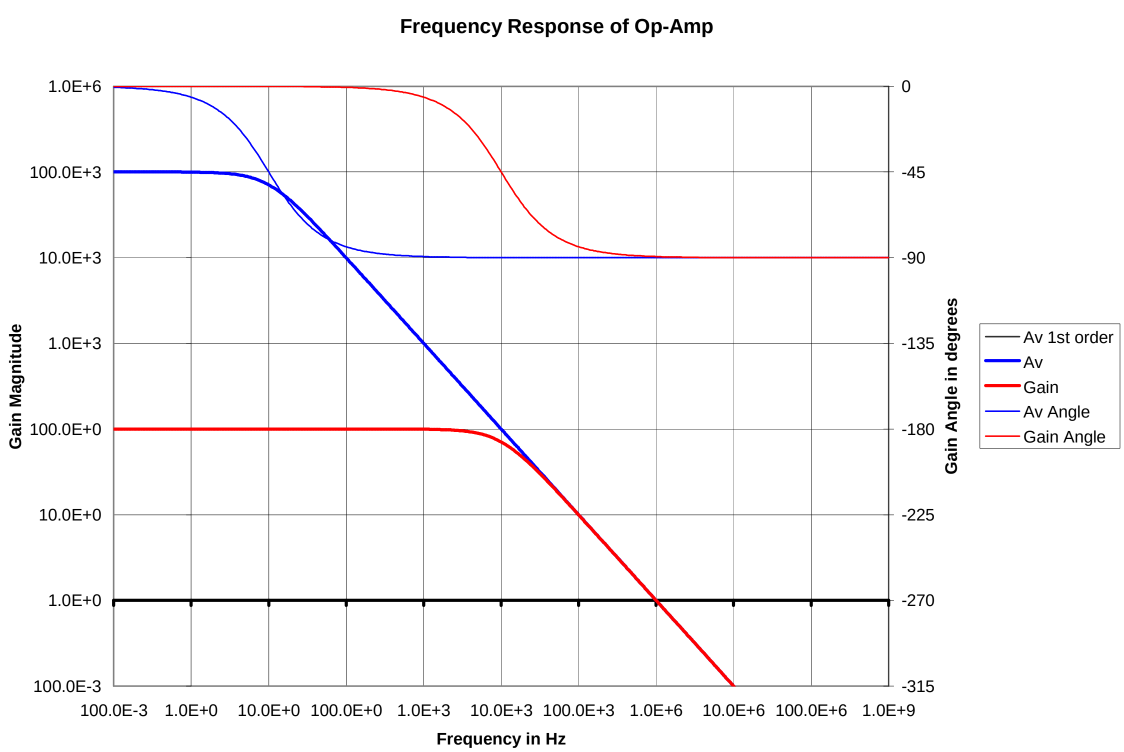

Figure 1: Example first-order op-amp frequency response

Op-Amp Frequency Response

Observe in Figure 1 that the unity gain frequency is 1.0 MHz and that the open-loop gain at very low frequencies is 100,000. The closed loop gain of 100 is good up to around 10 kHz. Observe also that the -45 degree phase angle (i.e. for the first order cut-off frequencies) on the open-loop gain occurs at 10 Hz and at 10 kHz for the closed loop gain. Note that the phase response for both open and closed loop situations goes to -90 degrees as expected with a first order model.

Since the cutoff frequency of a first order low-pass section is given by

$$Fc = \frac{1}{2 * \pi * T} \qquad \text{Eq. 3}$$

we can solve for T as

$$T = \frac{1}{2 * \pi * Fc} \qquad \text{Eq. 4}$$

and then noting that

$$Fc = \frac{GBW}{Av} \qquad \text{Eq. 5}$$

we can then solve for T1 as

$$T1 = \frac{1}{2 * \pi * (GBW / Av)} \qquad \text{Eq. 6}$$

Since Fc is typically around 10 Hz, then T1 is typically around 16 milliseconds.

Figure 2 shows the same system but with Av = 1,000,000. Notice that the open-loop cut-off frequency has dropped to 1 Hz but the overall closed-loop response is unchanged. This is the main reason that we are generally more interested in GBW that Av. However, it must be remembered that for gain accuracy (small errors are not visible on the log plots in this article) that Av should be about one hundred or more times the closed loop gain. It should be observed on the plots that gain accuracy at high frequencies is will suffer. That is ok as we generally only care about gain accuracy at low frequencies where it is easy to obtain.

Op-Amp Frequency Response

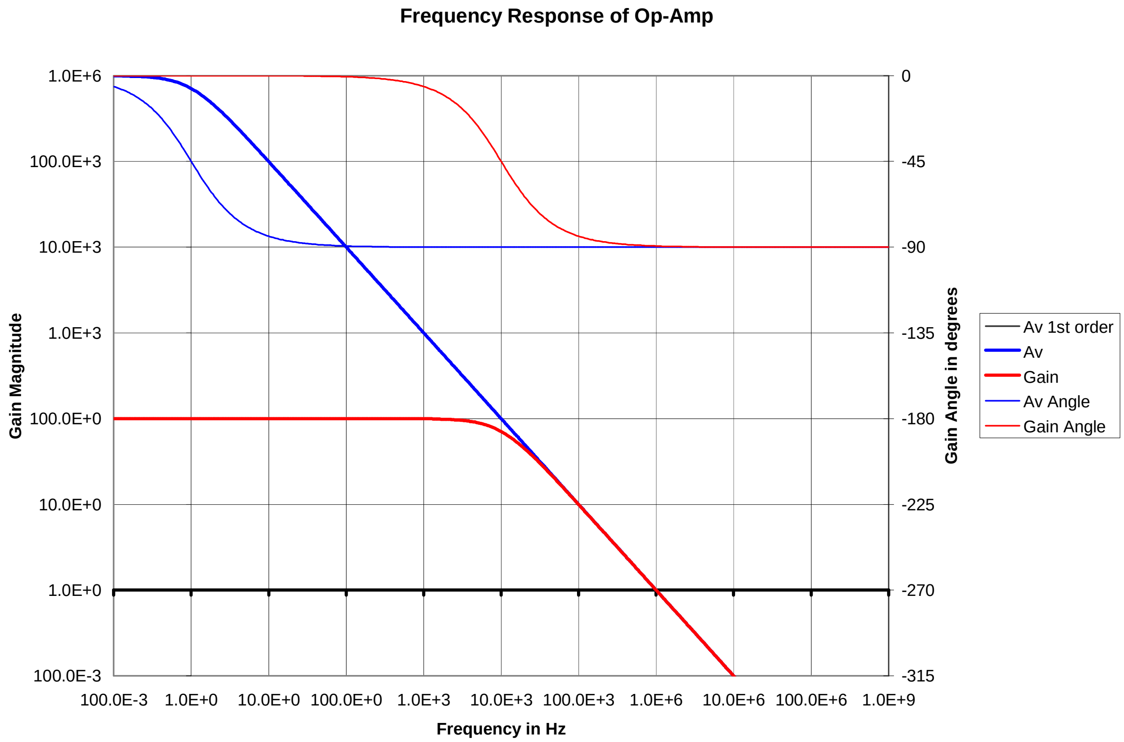

Figure 2: Same as Figure 1 but Av = 1,000,000

Note that the closed-loop cut-off frequency is 10 kHz which happens to the GBW of 1 MHz divided by the closed loop gain of 100. That is not a coincidence. We can predict the closed-loop bandwidth of an amplifier as follows.

$BW_{closed\_loop} = GBW / GAIN_{closed\_loop}$ Eq. 7

where GAIN is always the non-inverting gain – even if the amplifier is actually being used in inverting mode. There is a difference but for most cases it is too small to worry about unless the inverting gain is very low.

Figure 3 shows that in order to obtain higher closed-loop bandwidth that the GBW must be increased. For this example GBW is 10 MHz and Av has been reduced to 10,000. The closed loop bandwidth is now 100 kHz. The point of lowering Av is that it is common for higher bandwidth amplifiers to have relatively low open-loop DC gain. DC gain is generally not an issue in those applications so the op-amp designers do not go to the trouble of creating high DC gain. In fact, what is required for high DC gain tends to reduce GBW.

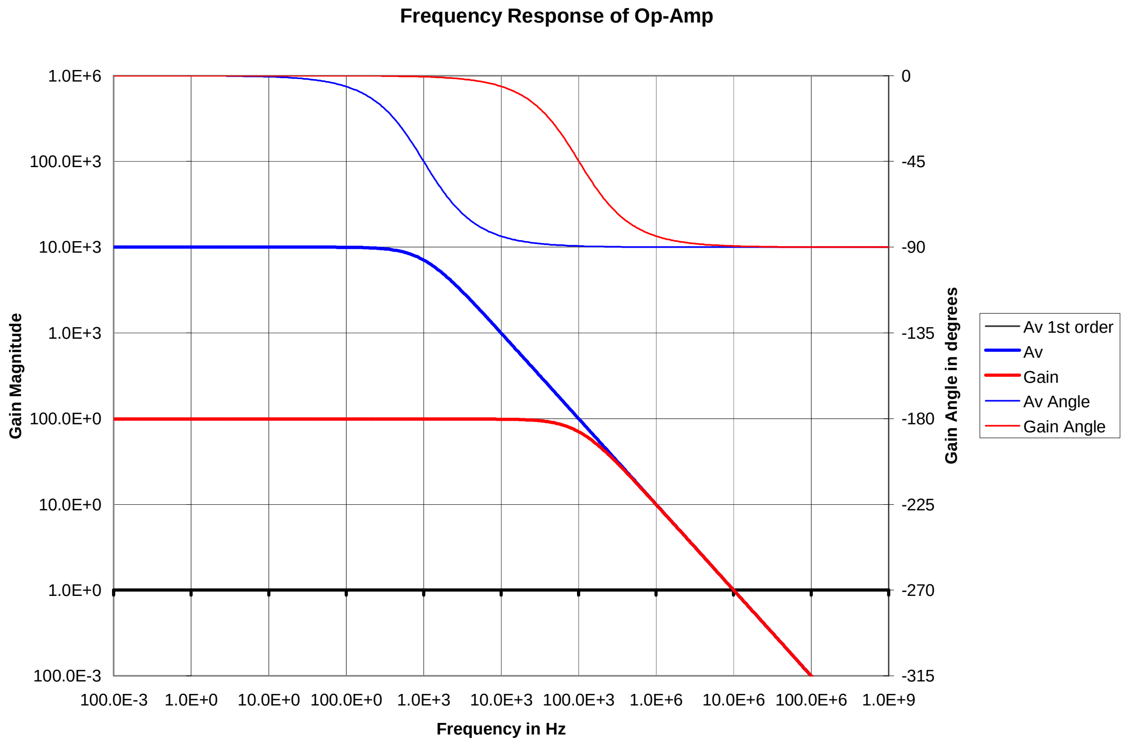

Figure 3: Increasing closed-loop bandwidth by increasing GBW

The final example shown in Figure 4 is taken from the previous but with the closed loop gain set to unity. As expected the bandwidth is 10 MHz. Notice that the response smoothly rolls off. That is not the case in reality. Things look fine here because we are using only a first order model. In reality, the op-amp has numerous poles in the transfer function and those will lead to some unexpected results as will be illustrated in the next sections.

Op-Amp Frequency Response

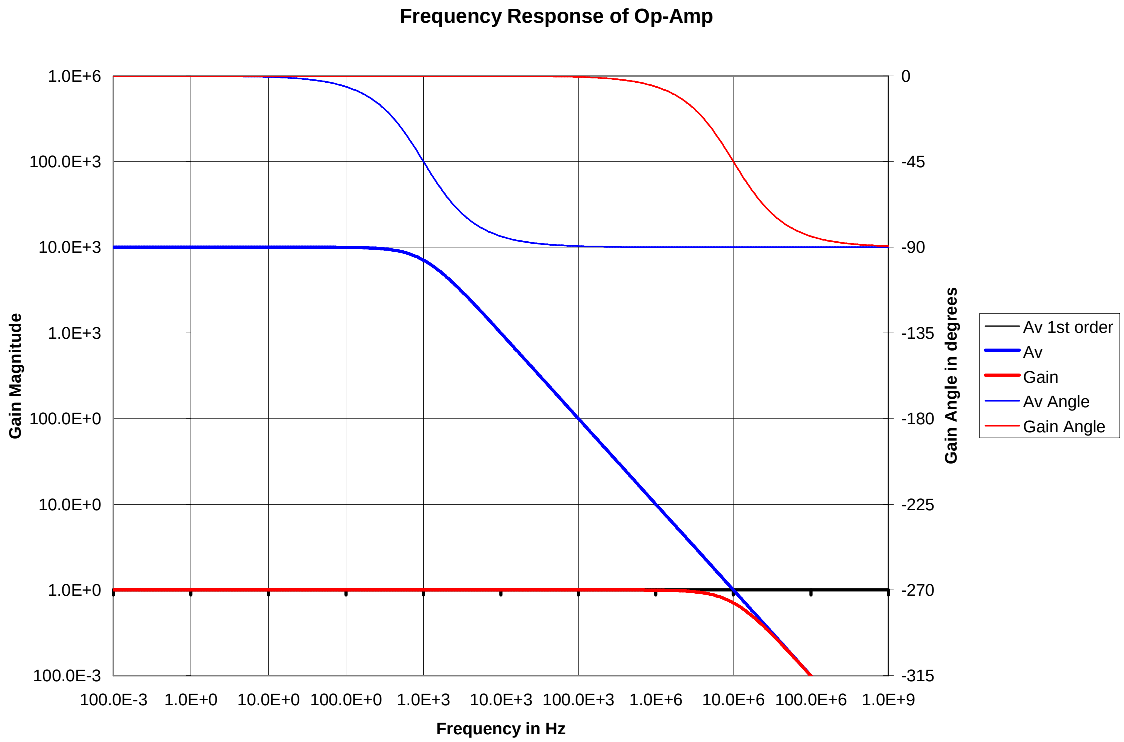

Figure 4: Unity closed-loop gain

Second order model of an op-amp

A second order model of an op-amp transfer function is shown below. Remember that T1 is the long time constant from the first order model that leads to a low cut-off frequency generally in the 10 Hz range. The time constant, T2, is much shorter and generally relates to a pole roughly one-tenth to one-half of GBW.

$$Vo = (Vin+ - Vin-) * Av * \frac{1}{T1*s + 1} * \frac{1}{T2*s + 1} \qquad \text{Eq. 8}$$

where:
T1 and T2 are time constants
s is jw

Figure 5 shows the response of an op-amp with a GBW of 10 MHz and a break frequency due to T2 of 1 MHz. You can see the extrapolated first order GBW of 10 MHz and that the actual unity gain frequency is lower. The gain slope becomes steeper at 1 MHz – now decreasing by a factor of 100 for each factor of 10 increase in frequency. With a second order model the phase angle increases to -180 degrees for both the open and closed loop responses. This is the start of a potential problem – an oscillator will result if the loop gain is 1.0 or more when the phase angle crosses 180 degrees. Figure 5 is a stable system as that criteria is not met.

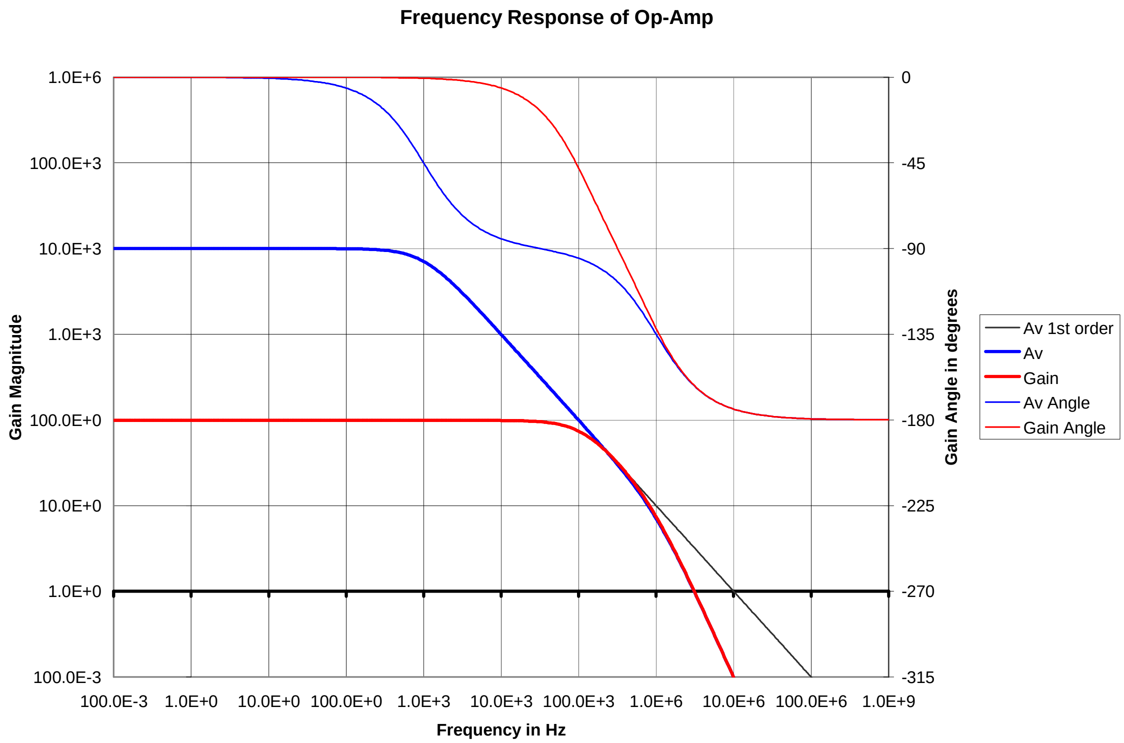

Figure 5: Second order op-amp model

Observe what happens in Figure 6 when the closed-loop gain is reduced to 1.0. Figure 6 suggests that response issues can occur at low gains. This is counter-intuitive as one generally thinks that low gain situations should be simple and easy. But the fact is that op-amp circuits can become unstable when the non-inverting gain is too low and compensation methods have not been applied. The instabilities at low gain can be compensated by using some advanced methods beyond the scope of this article.

Op-Amp Frequency Response

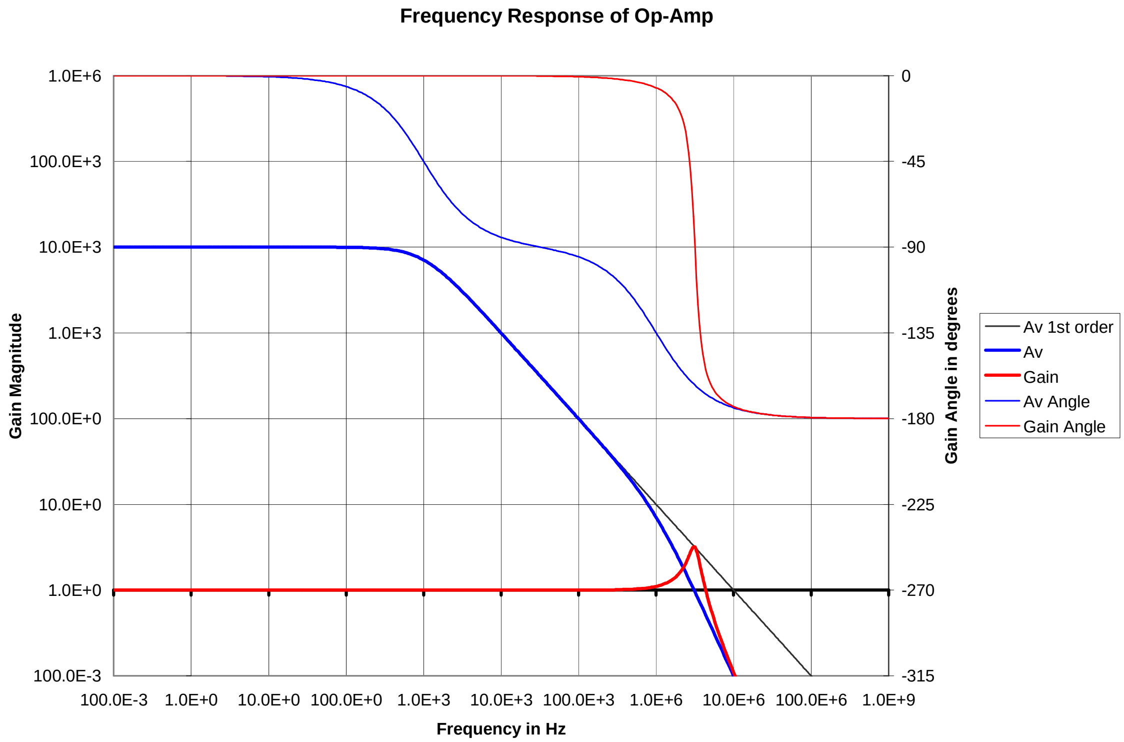

Figure 6: Closed-loop gain of 1.0 with peaking resulting from second-order effects

Third-order model of op-amp

A second order model suggests that problems can occur. A third and higher order model brings out the problems. A third order model of an op-amp is shown below.

$$Vo = (Vin+ - Vin-) * Av * \frac{1}{T1*s + 1} * \frac{1}{T2*s + 1} * \frac{1}{T3*s + 1} \qquad \text{Eq. 8}$$

where:
T1, T2, and T3 are time constants
s is jw

The T1 and T2 time constants are the same as used previously. The new time constant, T3, generally relates to a pole (or cut-off frequency) that is probably very roughly somewhere between a third and three-fourths the frequency distance between F2 (resulting from T2) and GBW. But this varies considerably depending on the design of the op-amp. In reality the order goes higher than three but our discussion will stop at three because that probably explains over 95 percent of the op-amp frequency response.

Figure 7 shows the previous unity gain example but with a new break frequency, F3, of 9 MHz. Note the extreme magnitude peak at around 3 MHz. Looking at the open-loop gain and phase response of the op-amp, it appears that the circuit is on the verge of oscillating if it actually isn't (coarseness in the frequency sample points on the plot may hide an oscillation). Oscillation will occur if the loop gain is greater than 1.0 when the phase crosses 180 degrees. Note that the phase response now goes to -270 degrees.

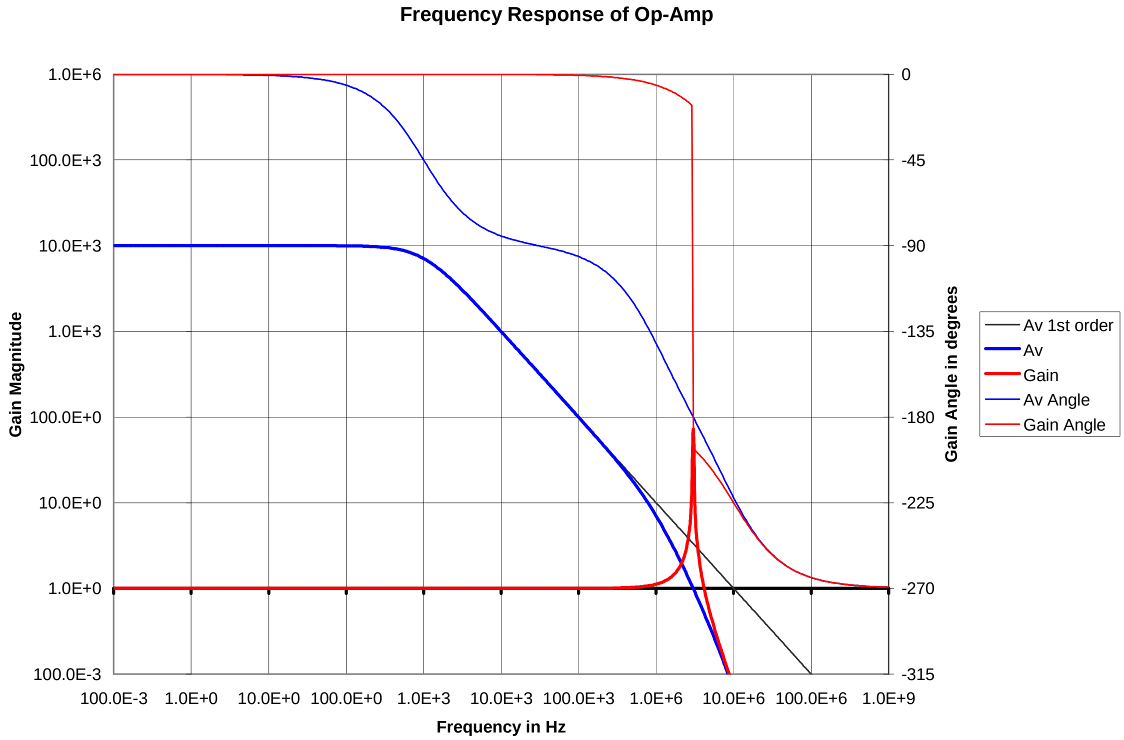

Figure 7: Unity gain amplifier with third-order op-amp model – oscillation likely

Figure 8 shows the earlier example of a gain of 100 with the third-order model. Note that the amplifier is stable since the loop gain at the 180 degree crossover is approximately 1.0 * (1 / 100) or 0.01. The ~1.0 is from the plot and the gain is 100 thus the feedback factor is 0.01.

Op-Amp Frequency Response

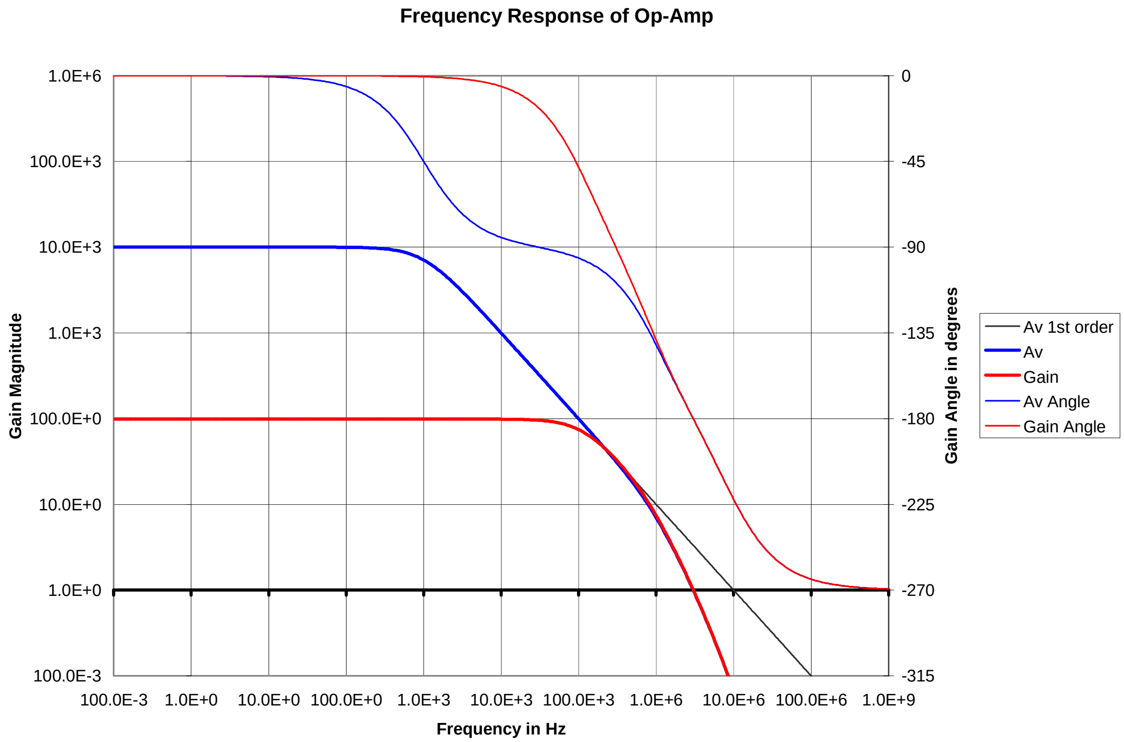

Figure 8: Gain of 100 with third-order model

Conclusions

The closed-loop frequency response of an amplifier is a strong function of the gain-bandwidth product of the amplifier. The gain-bandwidth product is also known as the unity gain frequency. As long as the amplifier gain is above roughly ten then a simple first order model gives very good results.

We can analyze for the bandwidth of an amplifier by knowing the closed-loop gain and the GBW of the op-amp as follows.

$$BW_{closed\_loop} = GBW / GAIN_{closed\_loop} \qquad \text{(Eq. 7)}$$

This is derived for a non-inverting amplifier but is applicable also to inverting amplifiers as there is little difference unless the gain is in the low single digits. This should be considered as an approximation as the actual GBW of the op-amp can vary from the typical value shown on the data sheet and GBW also varies with temperature, power supply voltage, and other effects.

For design we turn Equation 7 around and solve for the minimum value of GBW the op-amp must have given the gain and the desired bandwidth as follows. This equation also applies to inverting amplifiers too. Remember that this is only a good approximation.

$GBWmin = GAIN_{closed\_loop} * BW_{closed\_loop\_desired}$ Eq. 9

We will then choose an op-amp with a somewhat higher GBW than what we calculated to insure that the design specification is met. It is generally better to use an op-amp with only just enough GBW to accomplish our task. Using a very high GBW op-amp requires special physical construction of the circuit that is an unnecessary burden and can be the source of problems (oscillations as one example) that we would not have if we did not over-spec GBW.

Homework problems

Check your results using the spreadsheet – most problems are first order so set F2 and F3 to something like 1e12 so that they have no effect. An Av of 100,000 is fine – but play with that higher and lower and observe the effects.

1. An op-amp has a GBW of 1.5 MHz. What will be the bandwidth if the closed-loop gain is 50? (answer: 30 kHz)

2. It is desired to make an amplifier with a closed-loop gain of 10 and that has a bandwidth of 12 MHz. What is the minimum GBW for the op-amp? (answer: 120 MHz)

3. An op-amp has a GBW of 200 kHz. What will be the bandwidth if the closed-loop gain is 500? (answer: 400 Hz)

4. It is desired to make an audio amplifier with a closed-loop gain of 250 and that has a bandwidth of 20 kHz. What is the minimum GBW for the op-amp? (answer: 5 MHz)

5. Enter the following (next page) into the spreadsheet provided and observe the plot. Note that the phase angle of the open-loop response is -180 degrees at 10 MHz. Note that the gain at that frequency is somewhat over 2.0. This indicates that this op-amp is not unity gain stable. In fact, it will oscillate at approximately 10 MHz for any gain less than about 2. The amplifier is stable with the gain of 100. Set the gain to 1 and observe the plot. The bizarre phase slope near 10 MHz is an indication that oscillation is taking place (the coarse frequency increments of the spreadsheet hide the complete story here). Then try a gain of 1.5, 2.0, and 2.1. Note that the amplifier is stable (barely!) at a gain of 2.1 but the gain peaking is very high. Note that for some wide band op-amps the minimum gain for stability may be 5, 10 or even over 20! Experimentally determine the lowest closed loop gain that shows no peaking. (answer: ~22)

Op-Amp Frequency Response

50,000	Av, open-loop gain at DC
50.0E+6	Hz, GBW
5.0E+6	Hz, F2 break frequency
20.0E+6	Hz, F3 break frequency
100.00	non-inverting gain

For problem 5

Appendix: Notes about the spreadsheet

The spreadsheet converts the T*s + 1 term for each of the three sections into a vector, M at an angle A. The magnitude of the imaginary term is 2 * π * F * T. M is then the square root of the sum of the imaginary term squared plus the real term (i.e. 1.0) squared). The angle, A, is the arc-tangent of the ratio of the imaginary term to the real term.

The net vector in the denominator product is found by multiplying the three M terms and adding the three A terms.

The first order Av is found by dividing the DC Av by M1 at each frequency. The third order Av is found by dividing the DC Av by the composite vector magnitude, Mnet, at each frequency. The phase response of the op-amp is found by subtracting the composite vector phase, Anet, from zero at each frequency (remember the vector is in the denominator).

The closed-loop gain is computed as a vector and is developed as follows:

Vo = (Vin+ - Vin-) * ((Av/Mnet) at an angle of Anet)

Vin- = a * Vo where a is alpha and is the voltage divider feedback factor of the two resistors. Note that a is the reciprocal of the non-inverting gain. Thus we can write

$$\frac{Vo}{Vin} = \frac{[(Av/Mnet) \text{ at an angle of } Anet]}{1 + [(a * (Av/Mnet)) \text{ at an angle of } Anet]} \qquad \text{Eq. 9}$$

This is the same equation that would be used in the simple zero order case but now has vectors. In the simple case, Mnet would always be 1.0 and Anet would always be zero degrees.

Op-Amp Frequency Response

For computation purposes, we can simplify Equation 9 by dividing both numerator and denominator by (a * Av/Mnet). This gives

$$\frac{\mathrm{Vo}}{\mathrm{Vin}} = \frac{[\,(1/a) \text{ at an angle of } \mathrm{Anet}\,]}{(1 / (a * \mathrm{Av}/\mathrm{Mnet})) + [1.0 \text{ at an angle of } \mathrm{Anet}]} \qquad \text{Eq. 10}$$

Since the non-inverting gain, G, is 1/a as previously mentioned, we can write

$$\frac{\mathrm{Vo}}{\mathrm{Vin}} = \frac{[G \text{ at an angle of } \mathrm{Anet}]}{(G / (\mathrm{Av}/\mathrm{Mnet})) + [1.0 \text{ at an angle of } \mathrm{Anet}]} \qquad \text{Eq. 11}$$

This is the form used in the spreadsheet for computing the closed-loop frequency response.

The magnitude of the response is

$$\text{Magnitude} = \frac{G}{\mathrm{sqrt}\{[G / (\mathrm{Av}/\mathrm{Mnet})]\text{^}2 + [1.0 * \cos(\mathrm{Anet})]\text{^}2\}} \qquad \text{Eq. 12}$$

The phase angle is

$$\text{Angle} = \mathrm{Anet} - \mathrm{atan2}\{[G / (\mathrm{Av}/\mathrm{Mnet}) + \cos(\mathrm{Anet})]\ ,\ [\sin(\mathrm{Anet})]\} \qquad \text{Eq. 13}$$

The atan2(x,y) function returns the correct phase angle for the imaginary (y) component and the real (x) component.

The frequency has uniform exponential spacing with 50 points per decade from 0.1 Hz to 1 GHz.