SALLEN-KEY UNITY GAIN LOW-PASS FILTER

June 4, 1998
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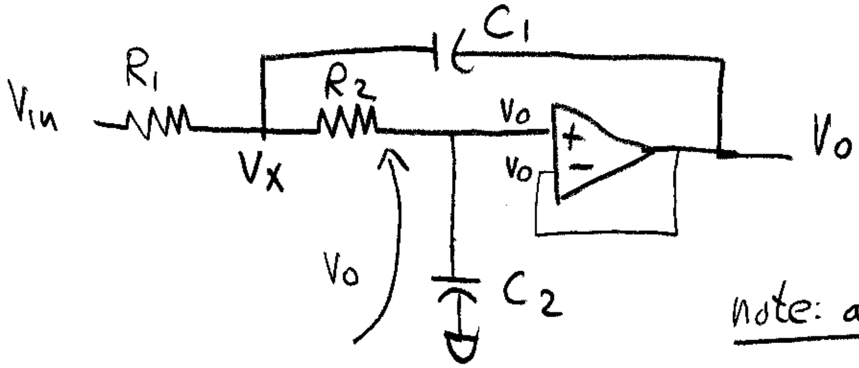

<u>note: all V's are V(s)</u>

$$\frac{V_{in}-V_x}{R_1} = \frac{V_x - V_o}{R_2} + \frac{V_x - V_o}{1/C_1 s}$$

$$R_2(V_{in}-V_x) = R_1(V_x - V_o) + R_1 R_2 C_1 s (V_x - V_o)$$

$$V_o = \frac{V_x}{R_2 C_2 s + 1} \qquad \therefore\ V_x = V_o(R_2 C_2 s + 1)$$

$$R_2\left(V_{in} - V_o(R_2 C_2 s + 1)\right) = R_1 V_o (R_2 C_2 s) + R_1 R_2 C_1 V_o (R_2 C_2 s)$$

$$V_o(R_1 C_2 s + R_1 C_1 R_2 C_2 s^2 + R_2 C_2 s + 1) = V_{in}$$

$$\frac{V_o(s)}{V_{in}(s)} = \frac{1}{R_1 C_1 R_2 C_2 s^2 + (R_1 + R_2) C_2 s + 1}$$

$$= \frac{1/R_1 C_1 R_2 C_2}{s^2 + \frac{R_1 + R_2}{R_1 R_2 C_1} s + \frac{1}{R_1 C_1 R_2 C_2}}$$

$$= \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\boxed{\omega_n = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}}$$

$$2\zeta\omega_n = \frac{R_1 + R_2}{R_1 R_2 C_1}$$

Analytic Equations

$$\frac{2\zeta}{\sqrt{R_1 R_2 C_1 C_2}} = \frac{R_1 + R_2}{R_1 R_2 C_1}$$

$$\zeta = \left(\frac{1}{2}\right)\left(\frac{R_1 + R_2}{R_1 R_2 C_1}\right)\sqrt{R_1 R_2 C_1 C_2}$$

$$\boxed{\zeta = \frac{(R_1 + R_2)}{2}\sqrt{\frac{C_2}{R_1 R_2 C_1}}}$$

Analysis: Given R_1, R_2, C_1, C_2 Find ω_n, ζ

Design: Given ω_n, ζ Find R_1 R_2 C_1 C_2

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there are infinite design solutions possible but only a limited set is practical. Generally:

* R_1 OR R_2 should be greater than about 1k
* R_1 OR R_2 should be less than about 1Meg
* C_1 OR C_2 should be greater than about 300 pF
* C_1 OR C_2 should be less than about 1uF

there are a number of possible design solutions.

A good example of a practical approch is contained within the spreadsheet sk_des.xls. The student will have to derive the math which is rather complicated — typical of design math.

SALLEN-KEY LPF DESIGN

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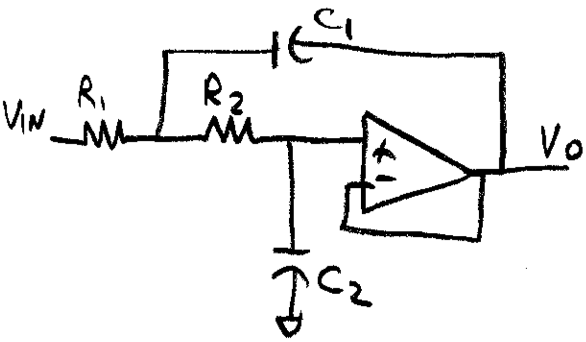
FROM ANALYSIS:



$$\omega_n = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}$$

$$\zeta = \frac{R_1 + R_2}{2}\sqrt{\frac{C_2}{R_1 R_2 C_1}}$$

WE CAN WRITE:

$$\zeta^2 = \frac{(R_1+R_2)^2}{4R_1R_2}\left(\frac{C_2}{C_1}\right) = \frac{R_1^2 + 2R_1R_2 + R_2^2}{4R_1R_2}\left(\frac{C_2}{C_1}\right)$$

$$= \left(\frac{R_1}{4R_2} + \frac{1}{2} + \frac{R_2}{4R_1}\right)\left(\frac{C_2}{C_1}\right)$$

$$= \left(\frac{1}{4(R_2/R_1)} + \frac{1}{2} + \frac{(R_2/R_1)}{4}\right)\left(\frac{C_2}{C_1}\right)$$

$$\left(\frac{R_2}{R_1}\right)\zeta^2 = \left(\frac{1}{4} + \frac{R_2/R_1}{2} + \frac{(R_2/R_1)^2}{4}\right)\left(\frac{C_2}{C_1}\right)$$

$$\left(\frac{C_2/C_1}{4}\right)\left(\frac{R_2}{R_1}\right)^2 + \left(\frac{C_2/C_1}{2} - \zeta^2\right)\left(\frac{R_2}{R_1}\right) + \frac{C_2/C_1}{4} = 0$$

$$\frac{R_2}{R_1} = \frac{\zeta^2 - \frac{C_2/C_1}{2} + \sqrt{\left(\frac{C_2/C_1}{2} - \zeta^2\right)^2 - \frac{(C_2/C_1)^2}{4}}}{\left(\frac{C_2/C_1}{4}\right)}$$

$$\frac{R_2}{R_1} = \frac{2\zeta^2 - \frac{C_2}{C_1} + 2\sqrt{\left[\cancel{\frac{(C_2/C_1)^2}{4}} - \frac{2(C_2/C_1)\zeta^2}{2} + \zeta^4\right] - \cancel{\frac{(C_2/C_1)^2}{4}}}}{(C_2/C_1)}$$

$$\boxed{\frac{R_2}{R_1} = \frac{2\zeta^2 - \left(\frac{C_2}{C_1}\right) + 2\zeta\sqrt{\zeta^2 - \left(\frac{C_2}{C_1}\right)}}{(C_2/C_1)}}$$

NOTE:

$\frac{C_2}{C_1} \leq \zeta^2$ for real sqrt

PROCEDURE:

1. DETERMINE $C = \sqrt{C_1 C_2} = \frac{4 \times 10^{-7}}{\sqrt{\text{Frequency (Hz)}}}$ Farads
2. $C_1 = C/\zeta$ and round C_1 to near std value
3. $C_2 < \zeta^2 C_1$ and round C_2 down to convenient std value
4. $K = \frac{R_2}{R_1} = \frac{2\zeta^2 - \left(\frac{C_2}{C_1}\right) + 2\zeta\sqrt{\zeta^2 - \frac{C_2}{C_1}}}{C_2/C_1}$
5. $R^2 = R_1 R_2 = \frac{1}{\omega_n^2 C_1 C_2}$
6. $K R_1^2 = R^2 \quad \therefore \quad R_1 = \sqrt{\frac{R^2}{K}}$ round to closest std value
7. $R_2 = \frac{R^2}{R_1}$ round to closest std value