Frequency Filter Notes
Ken Kuhn hand notes

Contents:

BUTTERWORTH
LOW-PASS FILTER

THE NORMALIZED RESPONSE OF THE FILTER IS GIVEN BY

$$R = \left| \frac{A_{vf}}{A_{vo}} \right| = \frac{1}{\sqrt{1 + \left(\frac{f}{f_c}\right)^{2n}}}$$

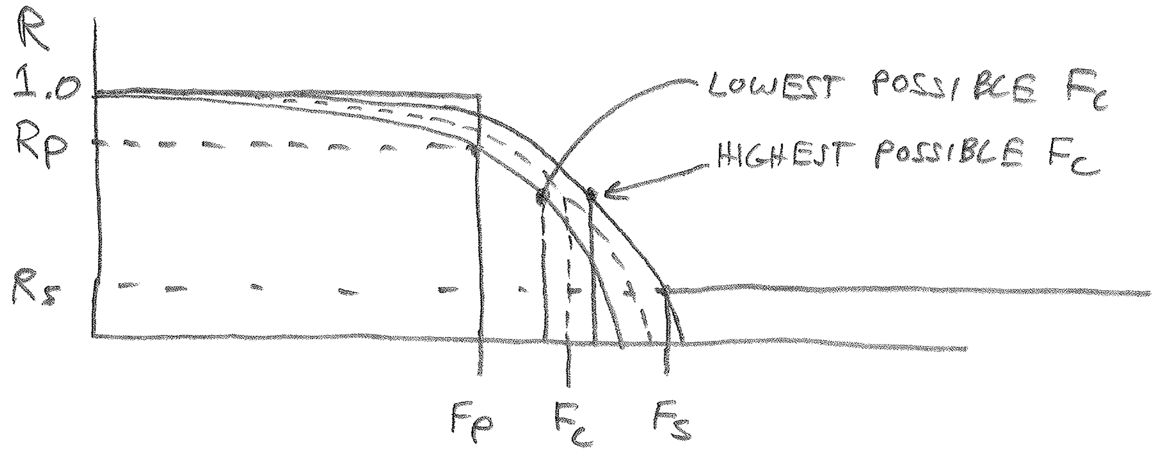
f = FREQUENCY OF INTEREST

f_c = CUTOFF FREQUENCY $\left(R = \frac{1}{\sqrt{2}} = .707\right)$

A_{vf} = RESPONSE AT FREQUENCY, f

A_{vo} = RESPONSE AT DC (TYPICALLY 1.0)

n = FILTER ORDER (n = 1, 2, 3, ---)



R_p = SMALLEST ALLOWABLE RESPONSE IN PASSBAND
R_p TYPICALLY IN THE RANGE 0.75 TO 0.95

R_s = LARGEST ALLOWABLE RESPONSE IN STOPBAND
R_s TYPICALLY LESS THAN 0.03

F_p = HIGHEST FREQUENCY IN PASSBAND

F_s = LOWEST FREQUENCY IN STOPBAND

F_c = CUTOFF FREQUENCY $R = \frac{1}{\sqrt{2}} = .707$

F_C IS GENERALLY THE AVERAGE OF THE LOWEST POSSIBLE F_{C_L} THAT JUST MEETS THE PASSBAND SPEC AND THE HIGHEST POSSIBLE F_{CH} THAT JUST MEETS THE STOPBAND SPEC. THE TWO CUTOFF FREQUENCIES ARISE BECAUSE OF INTEGER ROUNDING IN THE COMPUTATION OF n.

THE DESIGN PROBLEM IS:

GIVEN: R_P, R_S, F_P, F_S

FIND: n, F_C

$$\left.\begin{aligned} R_S &= \frac{1}{\sqrt{1+\left(\frac{F_S}{F_C}\right)^{2n}}} \\ R_P &= \frac{1}{\sqrt{1+\left(\frac{F_P}{F_C}\right)^{2n}}} \end{aligned}\right\} \text{SOLVE FOR } n, F_C$$

$$\left.\begin{aligned} R_s^2 &= \frac{1}{1+\left(\frac{F_s}{F_c}\right)^{2n}} \\ R_p^2 &= \frac{1}{1+\left(\frac{F_p}{F_c}\right)^{2n}} \end{aligned}\right\}$$

BASIC SET OF EQUATIONS

WE FIRST SOLVE FOR n THEN FOR F_c

$$\left(\frac{F_s}{F_c}\right)^{2n} = \frac{1}{R_s^2} - 1$$

$$\left(\frac{F_p}{F_c}\right)^{2n} = \frac{1}{R_p^2} - 1$$

$$\left(\frac{F_s}{F_p}\right)^{2n} = \frac{\frac{1}{R_s^2} - 1}{\frac{1}{R_p^2} - 1}$$

$$2n \ln\left(\frac{F_s}{F_p}\right) = \ln\left(\frac{\frac{1}{R_s^2} - 1}{\frac{1}{R_p^2} - 1}\right)$$

$$\boxed{n = \frac{\ln\left(\frac{\frac{1}{R_s^2} - 1}{\frac{1}{R_p^2} - 1}\right)}{2 \ln\left(\frac{F_s}{F_p}\right)}}$$

$\boxed{n = \text{integer}(n+1)}$ Round n up to next integer

THE LOWEST POSSIBLE CUTOFF FREQUENCY IS FOUND BY SOLVING:

$$R_p = \frac{1}{\sqrt{1 + \left(\frac{F_p}{F_{cL}}\right)^{2n}}} \qquad \text{FOR } F_{cL}$$

$$1 + \left(\frac{F_p}{F_{cL}}\right)^{2n} = \frac{1}{R_p^2}$$

$$2n \ln\left(\frac{F_p}{F_{cL}}\right) = \ln\left(\frac{1}{R_p^2} - 1\right)$$

$$\frac{F_p}{F_{cL}} = e^{\left[\ln\left(\frac{1}{R_p^2} - 1\right)/2n\right]}$$

$$\boxed{F_{cL} = \frac{F_p}{e^{\left[\ln\left(\frac{1}{R_p^2} - 1\right)/2n\right]}}}$$

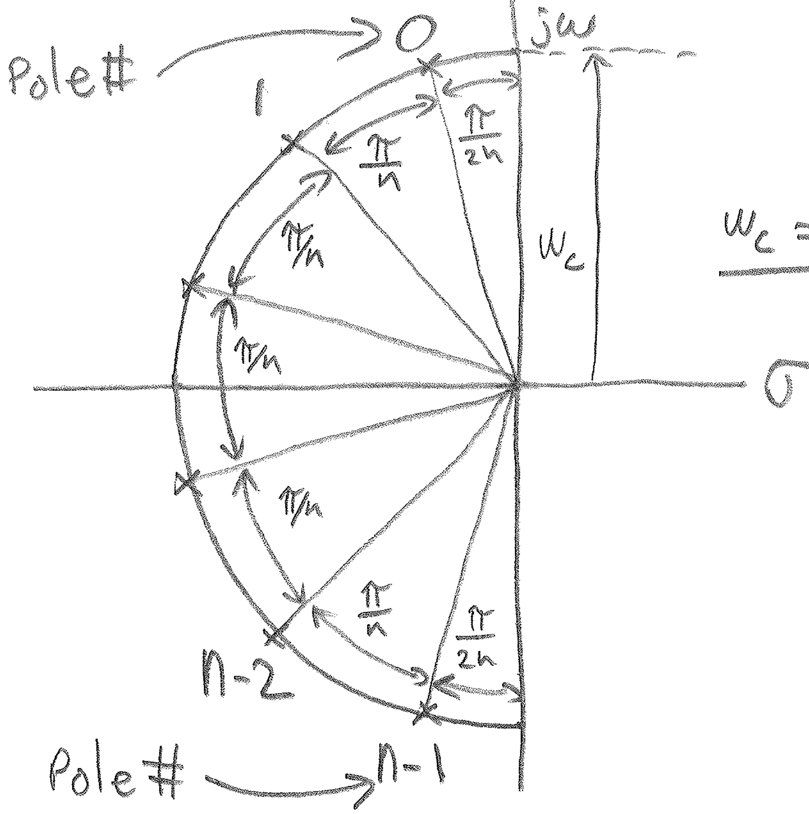
THE HIGHEST POSSIBLE CUTOFF FREQUENCY IS FOUND BY SOLVING

$$R_s = \frac{1}{\sqrt{1 + \left(\frac{F_s}{F_{cH}}\right)^{2n}}} \qquad \text{FOR } F_{cH}$$

$$\boxed{F_{cH} = \frac{F_s}{e^{\left[\ln\left(\frac{1}{R_s^2} - 1\right)/2n\right]}}}$$

$$\boxed{F_c = \frac{(F_{cL} + F_{cH})}{2}}$$

THE BUTTERWORTH POLE LOCATIONS ARE

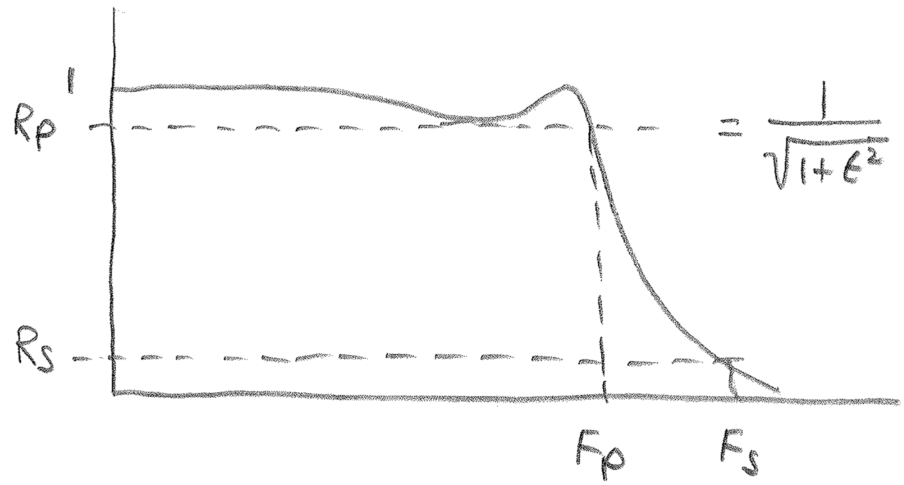


$$\omega_c = 2\pi F_c$$

$$\left.\begin{aligned} \sigma_k &= \omega_c \cos\left(\frac{\pi}{2}\left(1+\left(\frac{2k+1}{n}\right)\right)\right) \\ \omega_k &= \omega_c \sin\left(\frac{\pi}{2}\left(1+\left(\frac{2k+1}{n}\right)\right)\right) \end{aligned}\right\} k = 0, 1, \ldots, n-1$$

CHEBYSHEV LOW PASS FILTER

Jan 26, 98
K. KUHN



$$= \frac{1}{\sqrt{1+\epsilon^2}}$$

① $\epsilon = \sqrt{\frac{1}{R_p^2} - 1}$

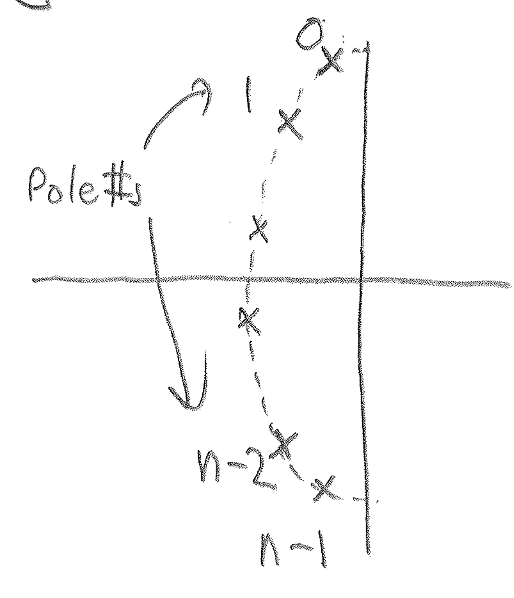
② $$n = \frac{\cosh^{-1}\left[\sqrt{\left(\frac{1}{R_s^2} - 1\right)} \Big/ \epsilon\right]}{\cosh^{-1}\left(F_s / F_p\right)}$$

Round n up to next higher integer

$$\text{pole}(k) = \omega_p \cos\left[\frac{\pi}{2}\left(1 + \frac{2k+1}{n}\right)\right] \sinh\left(\frac{1}{n}\sinh^{-1}\left(\frac{1}{\epsilon}\right)\right)$$
$$\pm j\,\omega_p \sin\left[\frac{\pi}{2}\left(1 + \frac{2k+1}{n}\right)\right] \cosh\left(\frac{1}{n}\sinh^{-1}\left(\frac{1}{\epsilon}\right)\right)$$

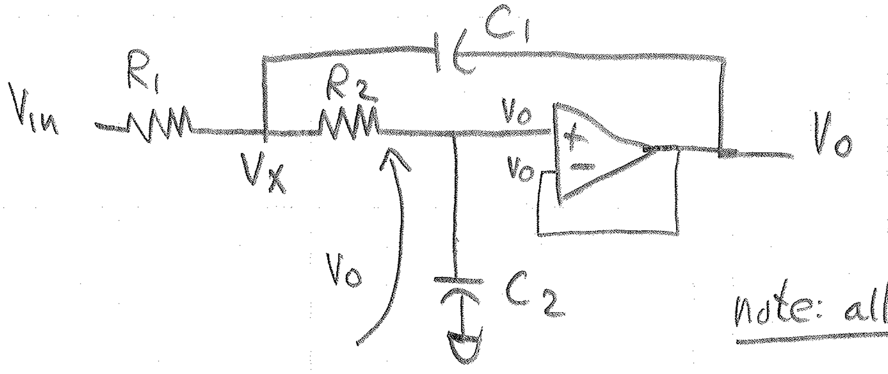
for $k = 0, 1, 2, \ldots n-1$
Save time using symmetry!

Poles located on ellipse



SALLEN-KEY UNITY GAIN LOW-PASS FILTER

June 4, 1998
K. Kuhn



note: all V's are V(s)

$$\frac{V_{in}-V_x}{R_1} = \frac{V_x - V_o}{R_2} + \frac{V_x - V_o}{1/C_1 s}$$

$$R_2(V_{in}-V_x) = R_1(V_x - V_o) + R_1R_2C_1 s(V_x - V_o)$$

$$V_o = \frac{V_x}{R_2C_2s+1} \qquad \therefore\ V_x = V_o(R_2C_2s+1)$$

$$R_2(V_{in} - V_o(R_2C_2s+1)) = R_1V_o(R_2C_2s) + R_1R_2C_1V_o(R_2C_2s)$$

$$V_o(R_1C_2s + R_1C_1R_2C_2s^2 + R_2C_2s + 1) = V_{in}$$

$$\frac{V_o(s)}{V_{in}(s)} = \frac{1}{R_1C_1R_2C_2s^2 + (R_1+R_2)C_2s + 1}$$

$$= \frac{1/R_1C_1R_2C_2}{s^2 + \frac{R_1+R_2}{R_1R_2C_1}s + \frac{1}{R_1C_1R_2C_2}}$$

$$= \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\boxed{\omega_n = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}}$$

$$2\zeta\omega_n = \frac{R_1 + R_2}{R_1 R_2 C_1}$$

Analytic Equations

$$\frac{2\zeta}{\sqrt{R_1 R_2 C_1 C_2}} = \frac{R_1 + R_2}{R_1 R_2 C_1}$$

$$\zeta = \left(\frac{1}{2}\right)\left(\frac{R_1 + R_2}{R_1 R_2 C_1}\right)\sqrt{R_1 R_2 C_1 C_2}$$

$$\boxed{\zeta = \frac{(R_1 + R_2)}{2}\sqrt{\frac{C_2}{R_1 R_2 C_1}}}$$

Analysis: Given R_1, R_2, C_1, C_2 Find ω_n, ζ

Design: Given ω_n, ζ Find $R_1\ R_2\ C_1\ C_2$

there are infinite design solutions possible but only a limited set is practical. Generally:

* R_1 OR R_2 should be greater than about 1k
* R_1 OR R_2 should be less than about 1 Meg
* C_1 OR C_2 should be greater than about 300 pF
* C_1 OR C_2 should be less than about 1 uF

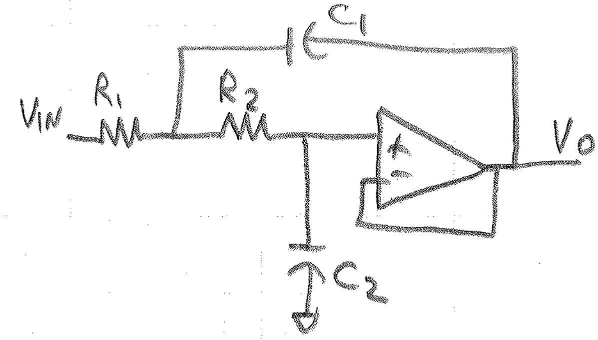
There are a number of possible design solutions. A good example of a practical approch is contained within the spreadsheet sk_des.xls. The student will have to derive the math which is rather complicated — typical of design math.

SALLEN-KEY LPF DESIGN

K. KUHN

FROM ANALYSIS:

$$\omega_n = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}$$



$$\zeta = \frac{R_1 + R_2}{2}\sqrt{\frac{C_2}{R_1 R_2 C_1}}$$

WE CAN WRITE:

$$\zeta^2 = \frac{(R_1+R_2)^2}{4R_1R_2}\left(\frac{C_2}{C_1}\right) = \frac{R_1^2 + 2R_1R_2 + R_2^2}{4R_1R_2}\left(\frac{C_2}{C_1}\right)$$

$$= \left(\frac{R_1}{4R_2} + \frac{1}{2} + \frac{R_2}{4R_1}\right)\left(\frac{C_2}{C_1}\right)$$

$$= \left(\frac{1}{4(R_2/R_1)} + \frac{1}{2} + \frac{(R_2/R_1)}{4}\right)\left(\frac{C_2}{C_1}\right)$$

$$\left(\frac{R_2}{R_1}\right)\zeta^2 = \left(\frac{1}{4} + \frac{R_2/R_1}{2} + \frac{(R_2/R_1)^2}{4}\right)\left(\frac{C_2}{C_1}\right)$$

$$\left(\frac{C_2/C_1}{4}\right)\left(\frac{R_2}{R_1}\right)^2 + \left(\frac{C_2/C_1}{2} - \zeta^2\right)\left(\frac{R_2}{R_1}\right) + \frac{C_2/C_1}{4} = 0$$

$$\frac{R_2}{R_1} = \frac{\zeta^2 - \frac{C_2/C_1}{2} + \sqrt{\left(\frac{C_2/C_1}{2} - \zeta^2\right)^2 - \frac{(C_2/C_1)^2}{4}}}{\left(\frac{C_2/C_1}{4}\right)}$$

$$\frac{R_2}{R_1} = \frac{2\zeta^2 - \frac{C_2}{C_1} + 2\sqrt{\left[\frac{(C_2/C_1)^2}{4} - \frac{2(C_2/C_1)\zeta^2}{2} + \zeta^4\right] - \frac{(C_2/C_1)^2}{4}}}{(C_2/C_1)}$$

$$\boxed{\frac{R_2}{R_1} = \frac{2\zeta^2 - \left(\frac{C_2}{C_1}\right) + 2\zeta\sqrt{\zeta^2 - \left(\frac{C_2}{C_1}\right)}}{(C_2/C_1)}}$$

NOTE:

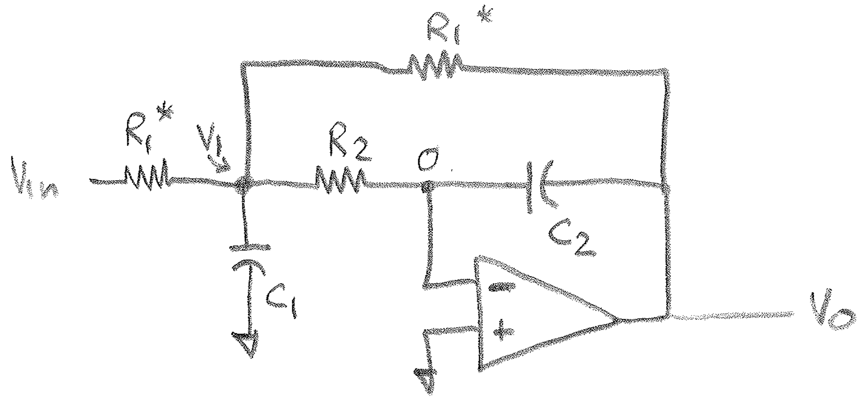
$\frac{C_2}{C_1} \leq \zeta^2$ for real sqrt

PROCEDURE:

1. DETERMINE $C = \sqrt{C_1 C_2} = \frac{4\times10^{-7}}{\sqrt{\text{Frequency (Hz)}}}$ Farads
2. $C_1 = C/\zeta$ and round C_1 to near std value
3. $C_2 < \zeta^2 C_1$ and round C_2 down to convenient std value
4. $K = \frac{R_2}{R_1} = \frac{2\zeta^2 - \left(\frac{C_2}{C_1}\right) + 2\zeta\sqrt{\zeta^2 - \frac{C_2}{C_1}}}{C_2/C_1}$
5. $R^2 = R_1 R_2 = \frac{1}{\omega_n^2 C_1 C_2}$
6. $K R_1^2 = R^2 \quad \therefore \quad R_1 = \sqrt{\frac{R^2}{K}}$ round to closest std value
7. $R_2 = \frac{R^2}{R_1}$ round to closest std value

INVERTING SECOND ORDER LOW-PASS FILTER

K. KUHN
1-23-00



* THE TWO RESISTORS ARE EQUAL FOR (INVERTED) UNITY GAIN AT DC

$$V_1 = \frac{V_{in}\,(R_1\|R_2)\|\frac{1}{C_1 s}}{R_1 + (R_1\|R_2)\|\frac{1}{C_1 s}} + \frac{V_o\,(R_1\|R_2)\|\frac{1}{C_1 s}}{R_1 + (R_1\|R_2)\|\frac{1}{C_1 s}}$$

$$(R_1\|R_2)\|\frac{1}{C_1 s} = \frac{\frac{R_1\|R_2}{C_1 s}}{R_1\|R_2 + \frac{1}{C_1 s}} = \frac{R_1\|R_2}{(R_1\|R_2)C_1 s + 1}$$

$$= \frac{R_1 R_2}{R_1 R_2 C_1 s + R_1 + R_2}$$

$$V_1 = \frac{(V_{in}+V_o)\ R_1 R_2}{R_1(R_1 R_2 C_1 s + R_1 + R_2) + R_1 R_2} = \frac{(V_{in}+V_o)\ R_2}{R_1 R_2 C_1 s + R_1 + 2R_2}$$

$$V_o = \frac{-V_1}{R_2 C_2 s} \qquad \therefore\ V_1 = -V_o R_2 C_2 s$$

$$-V_o R_2 C_2 s = \frac{(V_{in}+V_o)\,R_2}{R_1 R_2 C_1 s + R_1 + 2R_2}$$

$$\left(-V_o \cancel{R_2} C_2 s\right)\left(R_1 R_2 C_1 s + R_1 + 2R_2\right) = \left(V_{in} + V_o\right)\cancel{R_2}$$

$$-V_o\left[R_1 R_2 C_1 C_2 s^2 + \left(R_1 + 2R_2\right) C_2 s\right] = V_{in} + V_o$$

$$\frac{V_o}{V_{in}} = \frac{-1}{R_1 R_2 C_1 C_2 s^2 + \left(R_1 + 2R_2\right) C_2 s + 1}$$

$$= \frac{-1/R_1 R_2 C_1 C_2}{s^2 + \dfrac{\left(R_1 + 2R_2\right)\cancel{C_2} s}{R_1 R_2 C_1 \cancel{C_2}} + \dfrac{1}{R_1 R_2 C_1 C_2}}$$

$$\boxed{\frac{V_o}{V_{in}} = \frac{-1/R_1 R_2 C_1 C_2}{s^2 + \dfrac{\left(R_1 + 2R_2\right) s}{R_1 R_2 C_1} + \dfrac{1}{R_1 R_2 C_1 C_2}}} \Rightarrow \frac{-\omega_n^2}{s^2 + 2\zeta\omega_n + \omega_n^2}$$

(std. form)

$$\boxed{\omega_n = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}}$$

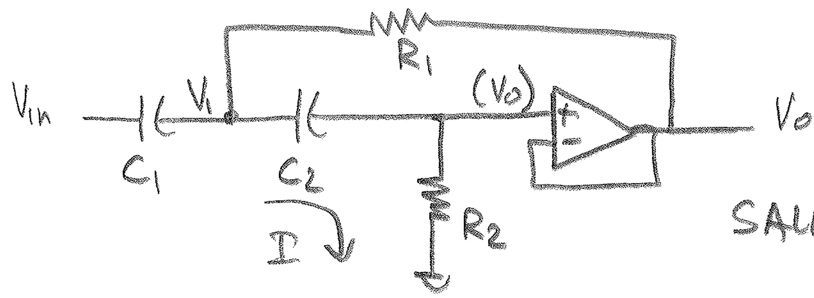
$$2\zeta\omega_n = \frac{R_1 + 2R_2}{R_1 R_2 C_1}$$

$$\zeta = \frac{\dfrac{R_1 + 2R_2}{R_1 R_2 C_1} * \sqrt{R_1 R_2 C_1 C_2}}{2} = \left(\frac{R_1}{2} + R_2\right)\sqrt{\frac{R_1 R_2 C_1 C_2}{R_1^2 R_2^2 C_1^2}}$$

$$= \left(\frac{R_1}{2} + R_2\right)\sqrt{\frac{C_2}{R_1 R_2 C_1}} \qquad \boxed{\zeta = \left(\frac{\dfrac{R_1}{2} + R_2}{\sqrt{R_1 R_2}}\right)\sqrt{\frac{C_2}{C_1}}}$$

NON-INVERTED SECOND ORDER HIGH-PASS FILTER

K. KUHN
1-23-00



SALLEN-KEY

$$\frac{V_{in}-V_1}{1/C_1 s} = \frac{V_1 - V_0}{R_1} + \frac{V_1 - V_0}{1/C_2 s}$$

$$I = \frac{V_0}{R_2}$$

$$V_1 = V_0 + \frac{I}{C_2 s} = V_0\left[1 + \frac{1}{R_2 C_2 s}\right] = V_0\left[\frac{R_2 C_2 s + 1}{R_2 C_2 s}\right]$$

$$\frac{V_{in} - V_0\left[\frac{R_2 C_2 s + 1}{R_2 C_2 s}\right]}{1/C_1 s} = \frac{V_0\left[\frac{R_2 C_2 s + 1}{R_2 C_2 s}\right] - V_0}{R_1} + \frac{V_0}{R_2}$$

$$V_0\left[\left[\frac{R_2 C_2 s + 1}{R_2 C_2 s}\right] C_1 s + \frac{R_2 C_2 s + 1}{R_1 R_2 C_2 s} - \frac{1}{R_1} + \frac{1}{R_2}\right] = V_{in} C_1 s$$

$$V_0\left[\frac{R_1 C_1 s(R_2 C_2 s + 1) + R_2 C_2 s + 1 - R_2 C_2 s + R_1 C_2 s}{R_1 R_2 C_2 s}\right] = V_{in} C_1 s$$

$$V_0\left[\frac{R_1 R_2 C_1 C_2 s^2 + R_1 C_1 s + \cancel{R_2 C_2 s} + 1 - \cancel{R_2 C_2 s} + R_1 C_2 s}{R_1 R_2 C_2 s}\right] = V_{in} C_1 s$$

$$\frac{V_o}{V_{in}} = \frac{R_1R_2C_1C_2s^2}{R_1R_2C_1C_2s^2 + R_1(C_1+C_2)s + 1}$$

$$\frac{V_o}{V_{in}} = \frac{s^2}{s^2 + \frac{\cancel{R_1}(C_1+C_2)s}{\cancel{R_1}R_2C_1C_2} + \frac{1}{R_1R_2C_1C_2}}$$

$$\boxed{\frac{V_o}{V_{in}} = \frac{s^2}{s^2 + \frac{(C_1+C_2)s}{R_2C_1C_2} + \frac{1}{R_1R_2C_1C_2}}} \Longrightarrow \frac{s^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

std form

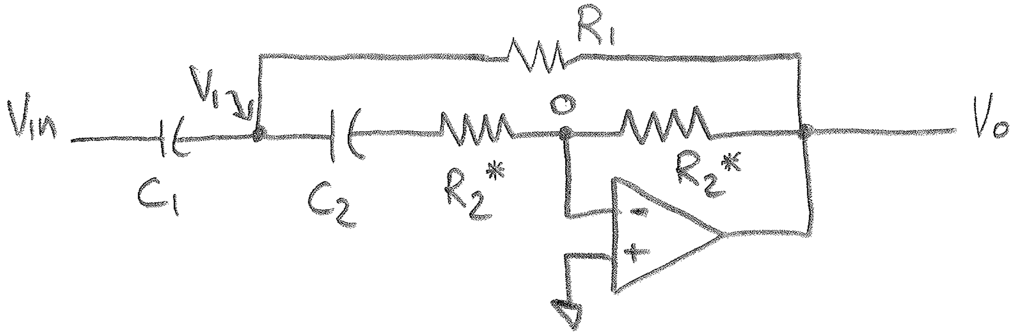
$$\boxed{\omega_n = \frac{1}{\sqrt{R_1R_2C_1C_2}}}$$

$$2\zeta\omega_n = \frac{(C_1+C_2)}{R_2C_1C_2}$$

$$\zeta = \frac{(C_1+C_2)\sqrt{R_1R_2C_1C_2}}{2R_2C_1C_2} = \boxed{\zeta = \frac{1}{2}\sqrt{\frac{R_1}{R_2}}\sqrt{\frac{(C_1+C_2)^2}{C_1C_2}}}$$

INVERTING SECOND ORDER HIGH-PASS FILTER

K. KUHN
1-23-00



* THE TWO RESISTORS ARE EQUAL FOR (INVERTED) UNITY GAIN AT HIGH FREQUENCIES

IMPEDANCE $\dfrac{1}{C_2 s} + R_2 = \dfrac{R_2 C_2 s + 1}{C_2 s}$

$$V_1 = \frac{V_{in}\left[R_1 \| \left(\frac{R_2C_2s+1}{C_2s}\right)\right]}{\frac{1}{C_1s} + R_1 \| \left(\frac{R_2C_2s+1}{C_2s}\right)} + \frac{V_o\left[\frac{1}{C_1s} \| \left(\frac{R_2C_2s+1}{C_2s}\right)\right]}{R_1 + \frac{1}{C_1s} \| \left(\frac{R_2C_2s+1}{C_2s}\right)}$$

$$R_1 \| \left(\frac{R_2C_2s+1}{C_2s}\right) = \frac{R_1\left(\frac{R_2C_2s+1}{C_2s}\right)}{R_1 + \frac{R_2C_2s+1}{C_2s}} = \frac{R_1(R_2C_2s+1)}{R_1C_2s + R_2C_2s+1}$$

$$= \frac{R_1(R_2C_2s+1)}{(R_1+R_2)C_2s+1}$$

$$\frac{1}{C_1s} \| \left(\frac{R_2C_2s+1}{C_2s}\right) = \frac{\frac{R_2C_2s+1}{C_1C_2s^2}}{\frac{1}{C_1s} + \frac{R_2C_2s+1}{C_2s}} = \frac{R_2C_2s+1}{C_2s + R_2C_1C_2s^2 + C_1s}$$

$$= \frac{R_2C_2s+1}{R_2C_1C_2s^2 + (C_1+C_2)s}$$

$$V_1 = \frac{\dfrac{V_{in} R_1 (R_2C_2s+1)}{(R_1+R_2)C_2s+1}}{\dfrac{1}{C_1s} + \dfrac{R_1(R_2C_2s+1)}{(R_1+R_2)C_2s+1}} + \frac{\dfrac{V_0(R_2C_2s+1)}{R_2C_1C_2s^2+(C_1+C_2)s}}{R_1 + \dfrac{R_2C_2s+1}{R_2C_1C_2s^2+(C_1+C_2)s}}$$

$$= \frac{V_{in} R_1(R_2C_2s+1)}{\dfrac{(R_1+R_2)C_2s+1}{C_1s} + R_1(R_2C_2s+1)} + \frac{V_0(R_2C_2s+1)}{R_1(R_2C_1C_2s^2+(C_1+C_2)s)+R_2C_2s+1}$$

$$= \frac{V_{in}R_1C_1s(R_2C_2s+1)}{(R_1+R_2)C_2s+1 + R_1C_1s(R_2C_2s+1)} + \frac{V_0(R_2C_2s+1)}{R_1(R_2C_1C_2s^2+(C_1+C_2)s)+R_2C_2s+1}$$

$$= \frac{V_{in}R_1C_1s(R_2C_2s+1) + V_0(R_2C_2s+1)}{R_1R_2C_1C_2s^2+(R_1C_1+R_1C_2+R_2C_2)s+1}$$

(DENOMINATORS ARE THE SAME WHEN EXPANDED)

$$V_0 = \frac{-V_1 R_2}{\dfrac{R_2C_2s+1}{C_2s}} = \frac{-V_1R_2C_2s}{R_2C_2s+1} \qquad \therefore V_1 = \frac{-V_0(R_2C_2s+1)}{R_2C_2s}$$

$$\frac{-V_0\cancel{(R_2C_2s+1)}}{R_2C_2s} = \frac{V_{in}R_1C_1s\cancel{(R_2C_2s+1)} + V_0\cancel{(R_2C_2s+1)}}{R_1R_2C_1C_2s^2+(R_1C_1+R_1C_2+R_2C_2)s+1}$$

$$-V_0\left[\frac{1}{R_2C_2s} + \frac{1}{R_1R_2C_1C_2s^2+(R_1C_1+R_1C_2+R_2C_2)s+1}\right] = \frac{V_{in}R_1C_1s}{R_1R_2C_1C_2s^2+(R_1C_1+R_1C_2+R_2C_2)s+1}$$

$$-V_0\left[1 + \frac{R_2C_2s}{R_1R_2C_1C_2s^2+(R_1C_1+R_1C_2+R_2C_2)s+1}\right] = \frac{V_{in}R_1R_2C_1C_2s^2}{R_1R_2C_1C_2s^2+(R_1C_1+R_1C_2+R_2C_2)s+1}$$

$$-V_0\left[\frac{R_1R_2C_1C_2s^2+(R_1C_1+R_1C_2+2R_2C_2)s+1}{\cancel{R_1R_2C_1C_2s^2+(R_1C_1+R_1C_2+R_2C_2)s+1}}\right] = \frac{V_1\ R_1R_2C_1C_2s^2}{\cancel{R_1R_2C_1C_2s^2+(R_1C_1+R_1C_2+R_2C_2)s+1}}$$

$$\frac{V_0}{V_{in}} = \frac{-R_1R_2C_1C_2s^2}{R_1R_2C_1C_2s^2+(R_1C_1+R_1C_2+2R_2C_2)s+1}$$

$$\boxed{\frac{V_0}{V_{in}} = \frac{-s^2}{s^2+\frac{(R_1C_1+R_1C_2+2R_2C_2)s}{R_1R_2C_1C_2}+\frac{1}{R_1R_2C_1C_2}}} \Rightarrow \frac{-s^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

std. form

$$\boxed{\omega_n = \frac{1}{\sqrt{R_1R_2C_1C_2}}}$$

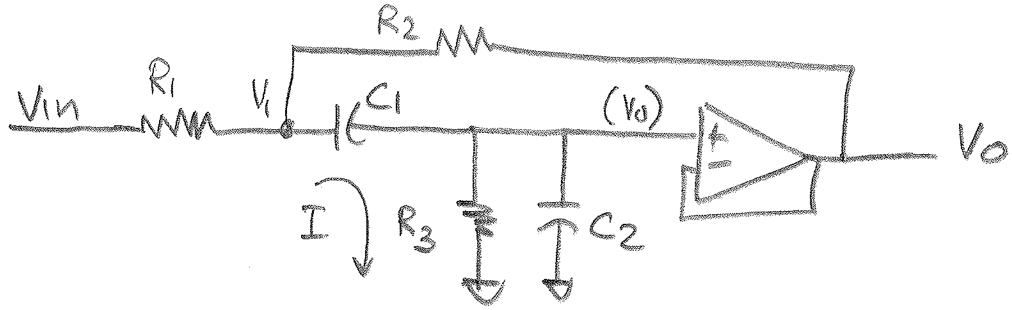
$$2\zeta\omega_n = \frac{(R_1C_1+R_1C_2+2R_2C_2)}{R_1R_2C_1C_2}$$

$$\zeta = \frac{(R_1C_1+R_1C_2+2R_2C_2)\sqrt{R_1R_2C_1C_2}}{2R_1R_2C_1C_2}$$

$$\boxed{\zeta = \frac{R_1C_1+R_1C_2+2R_2C_2}{2\sqrt{R_1R_2C_1C_2}}}$$

NON-INVERTING SECOND-ORDER BAND-PASS FILTER

K. KUHN
1-23-00



$$\frac{V_{in} - V_1}{R_1} = \frac{V_1 - V_O}{R_2} + \frac{V_1 - V_O}{1/C_1 s}$$

$$R_3 \| \frac{1}{C_2 s} = \frac{R_3/C_2 s}{R_3 + 1/C_2 s} = \frac{R_3}{R_3 C_2 s + 1}$$

$$I = \frac{V_O (R_3 C_2 s + 1)}{R_3}$$

$$V_1 = V_O + \frac{I}{C_1 s} = V_O\left[1 + \frac{R_3 C_2 s + 1}{R_3 C_1 s}\right] = V_O\left[\frac{R_3(C_1 + C_2)s + 1}{R_3 C_1 s}\right]$$

$$\frac{V_{in} - V_O\left[\frac{R_3(C_1+C_2)s+1}{R_3 C_1 s}\right]}{R_1} = \frac{V_O\left[\frac{R_3(C_1+C_2)s+1}{R_3 C_1 s}\right] - V_O}{R_2} + \frac{V_O(R_3 C_2 s + 1)}{R_3}$$

$$V_O\left[\frac{R_3(C_1+C_2)s+1}{R_1 R_3 C_1 s} + \frac{R_3(C_1+C_2)s+1}{R_2 R_3 C_1 s} - \frac{1}{R_2} + \frac{R_3 C_2 s + 1}{R_3}\right] = \frac{V_{in}}{R_1}$$

$$V_O\left[\frac{R_2 R_3(C_1+C_2)s + R_2 + R_1 R_3 (C_1+C_2)s + R_1 - R_1 R_3 C_1 s + R_1 R_2 R_3 C_1 C_2 s^2 + R_1 R_2 C_1 s}{\cancel{R_1} R_2 R_3 C_1 s}\right] = \frac{V_{in}}{\cancel{R_1}}$$

$$V_o\left[\frac{R_1R_2R_3C_1C_2s^2 + (R_2R_3C_1 + R_2R_3C_2 \cancel{+R_1R_3C_1} + R_1R_3C_2 \cancel{-R_1R_3C_1} + R_1R_2C_1)s + R_1 + R_2}{R_2R_3C_1s}\right] = V_{in}$$

$$\frac{V_o}{V_{in}} = \frac{R_2R_3C_1s}{R_1R_2R_3C_1C_2s^2 + (R_1R_2C_1 + R_1R_3C_2 + R_2R_3C_1 + R_2R_3C_2)s + R_1 + R_2}$$

$$= \frac{\cancel{R_2}\cancel{R_3}\cancel{C_1}s / R_1\cancel{R_2}\cancel{R_3}\cancel{C_1}C_2}{s^2 + \dfrac{(R_1R_2C_1 + R_1R_3C_2 + R_2R_3C_1 + R_2R_3C_2)s}{R_1R_2R_3C_1C_2} + \dfrac{R_1 + R_2}{R_1R_2R_3C_1C_2}}$$

$$\boxed{\frac{V_o}{V_{in}} = \frac{s/R_1C_2}{s^2 + \dfrac{(R_1R_2C_1 + R_1R_3C_2 + R_2R_3C_1 + R_2R_3C_2)s}{R_1R_2R_3C_1C_2} + \dfrac{R_1 + R_2}{R_1R_2R_3C_1C_2}}}$$

THE PEAK RESPONSE AT $\omega = \omega_n$

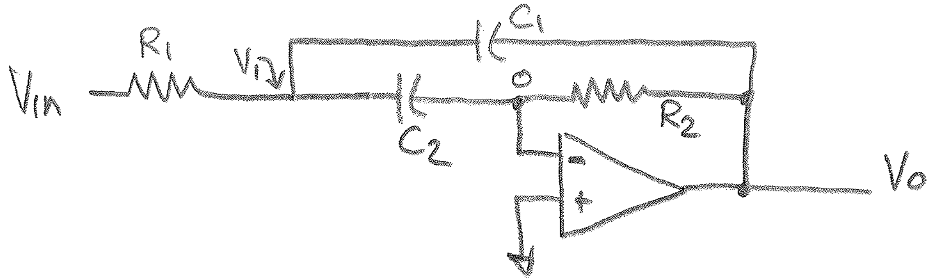
$$= \frac{1}{\cancel{R_1}\cancel{C_2}} \times \frac{\cancel{R_1}R_2R_3C_1\cancel{C_2}}{R_1R_2C_1 + R_1R_3C_2 + R_2R_3C_1 + R_2R_3C_2}$$

$$= \frac{1}{\dfrac{R_1}{R_2} + \dfrac{R_1C_2}{R_2C_1} + 1 + \dfrac{C_2}{C_1}}$$

$$= \frac{1}{\dfrac{R_1}{R_3} + \left(1 + \dfrac{R_1}{R_2}\right)\dfrac{C_2}{C_1}}$$

INVERTING SECOND ORDER BAND-PASS FILTER

K. KUHN
1-23-00



$$\frac{1}{C_1 s} \parallel \frac{1}{C_2 s} = \frac{\frac{1}{C_1 s}\frac{1}{C_2 s}}{\frac{1}{C_1 s}+\frac{1}{C_2 s}} = \frac{1}{(C_1+C_2)s}$$

$$R_1 \parallel \frac{1}{C_2 s} = \frac{\frac{R_1}{C_2 s}}{R_1+\frac{1}{C_2 s}} = \frac{R_1}{R_1 C_2 s+1}$$

$$V_1 = \frac{V_{in}\left(\frac{1}{(C_1+C_2)s}\right)}{R_1+\frac{1}{(C_1+C_2)s}} + \frac{V_o\left(\frac{R_1}{R_1 C_2 s+1}\right)}{\frac{1}{C_1 s}+\frac{R_1}{R_1 C_2 s+1}}$$

$$= \frac{V_{in}}{R_1(C_1+C_2)s+1} + \frac{V_o R_1}{\frac{R_1 C_2 s+1}{C_1 s}+R_1}$$

$$= \frac{V_{in}}{R_1(C_1+C_2)s+1} + \frac{V_o R_1 C_1 s}{R_1(C_1+C_2)s+1}$$

$$V_o = -V_1 R_2 C_2 s \quad \therefore \quad V_1 = \frac{-V_o}{R_2 C_2 s}$$

$$\frac{-V_o}{R_2 C_2 s} = \frac{V_{in}}{R_1(C_1+C_2)s+1} + \frac{V_o R_1 C_1 s}{R_1(C_1+C_2)s+1}$$

$$-V_o\left[\frac{1}{R_2C_2s} + \frac{R_1C_1s}{R_1(C_1+C_2)s+1}\right] = \frac{V_{in}}{R_1(C_1+C_2)s+1}$$

$$-V_o\left[1 + \frac{R_1R_2C_1C_2s^2}{R_1(C_1+C_2)s+1}\right] = \frac{V_{in}R_2C_2s}{R_1(C_1+C_2)s+1}$$

$$-V_o\left[\frac{R_1R_2C_1C_2s^2+R_1(C_1+C_2)s+1}{\cancel{R_1(C_1+C_2)s+1}}\right] = \frac{V_{in}R_2C_2s}{\cancel{R_1(C_1+C_2)s+1}}$$

$$\frac{V_o}{V_{in}} = \frac{-R_2C_2s}{R_1R_2C_1C_2s^2+R_1(C_1+C_2)s+1} = \frac{-R_2C_2s/R_1R_2C_1C_2}{s^2+\frac{R_1(C_1+C_2)s}{R_1R_2C_1C_2}+\frac{1}{R_1R_2C_1C_2}}$$

$$\boxed{\frac{V_o}{V_{in}} = \frac{-s/R_1C_1}{s^2+\frac{(C_1+C_2)s}{R_2C_1C_2}+\frac{1}{R_1R_2C_1C_2}}} \Rightarrow \frac{-2\zeta\omega_nKs}{s^2+2\zeta\omega_ns+\omega_n^2}$$

std. form

$$\boxed{\omega_n = \frac{1}{\sqrt{R_1R_2C_1C_2}}}$$

$$2\zeta\omega_n = \frac{C_1+C_2}{R_2C_1C_2}$$

$$\zeta = \frac{(C_1+C_2)\sqrt{R_1R_2C_1C_2}}{2R_2C_1C_2} = \boxed{\zeta = \frac{1}{2}\sqrt{\frac{R_1}{R_2}}\sqrt{\frac{(C_1+C_2)^2}{C_1C_2}}}$$

$$2\zeta\omega_nK = \frac{1}{R_1C_1}$$

$$K = \frac{\frac{1}{R_1C_1}}{\frac{(C_1+C_2)}{R_2C_1C_2}} = \frac{1}{R_1C_1}\cdot\frac{R_2C_1C_2}{C_1+C_2} = \frac{R_2C_2}{R_1(C_1+C_2)} = \boxed{K = \left(\frac{R_2}{R_1}\right)\left(\frac{1}{1+\frac{C_1}{C_2}}\right)}$$

THE PEAK MAGNITUDE RESPONSE AT $\omega = \omega_n$ IS $\underline{K}$

AN ALTERNATE REPRESENTATION OF K IS:

$$K = \frac{1}{R_1 C_1} \cdot \frac{R_2 C_1 C_2}{C_1 + C_2} = \frac{1}{C_1}\left(\frac{R_2}{R_1}\right)\left(\frac{C_1 C_2}{C_1 + C_2}\right)$$

NOTE THAT

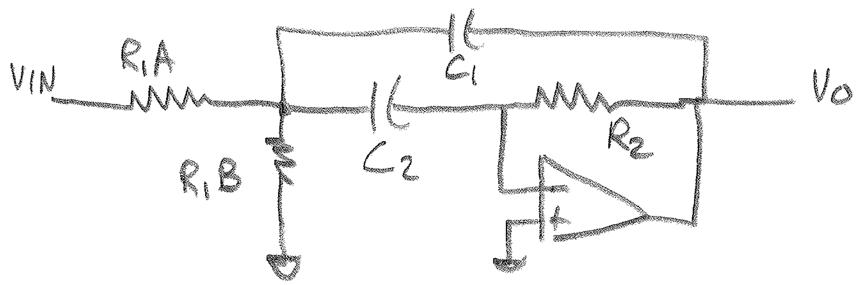
$$\zeta^2 = \frac{1}{4}\left(\frac{R_1}{R_2}\right)\left(\frac{(C_1 + C_2)^2}{C_1 C_2}\right)$$

THUS,

$$\boxed{K = \left(\frac{C_1 + C_2}{C_1}\right)\frac{1}{4\zeta^2} = \left(1 + \frac{C_2}{C_1}\right)\frac{1}{4\zeta^2} = \left(1 + \frac{C_2}{C_1}\right)Q^2}$$

$$\left(\zeta = \frac{1}{2Q}\right)$$

IT IS SOMETIMES DESIRABLE TO ATTENUATE THE TRANSFER FUNCTION SO THAT THE PEAK RESPONSE IS 1.0. THE CIRCUIT IS MODIFIED AS FOLLOWS:



$$R_1 = R_{1A} \| R_{1B}$$

$$\frac{1}{K} = \frac{R_{1B}}{R_{1A} + R_{1B}} = \frac{1}{1 + \frac{R_{1A}}{R_{1B}}}$$

$$\frac{R_{1A}}{R_{1B}} = K - 1 \qquad \therefore\ R_{1B} = \frac{R_{1A}}{K-1}$$

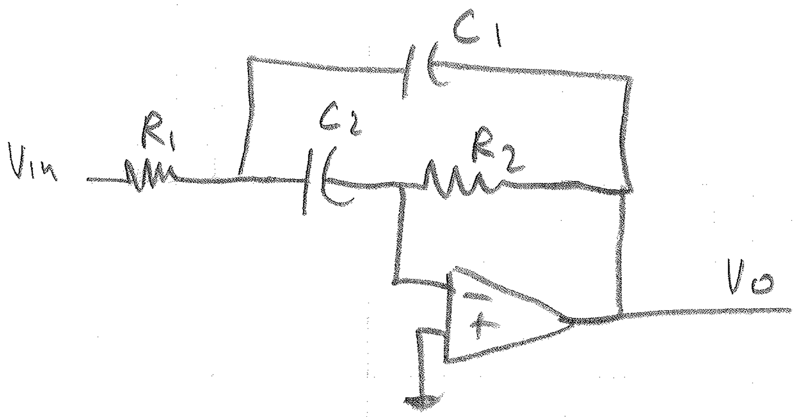
$$R_1 = \frac{R_{1A} R_{1B}}{R_{1A} + R_{1B}} = \frac{R_{1A}}{K+1}$$

$$\boxed{\begin{aligned} R_{1A} &= R_1(K+1) \\ R_{1B} &= \frac{R_{1A}}{K-1} \end{aligned}}$$

DESIGN NOTE FOR INVERTING BAND-PASS FILTER

JAN 23, 01

K. KUHN



$$F = \frac{1}{2\pi\sqrt{R_1 R_2}\, C}$$

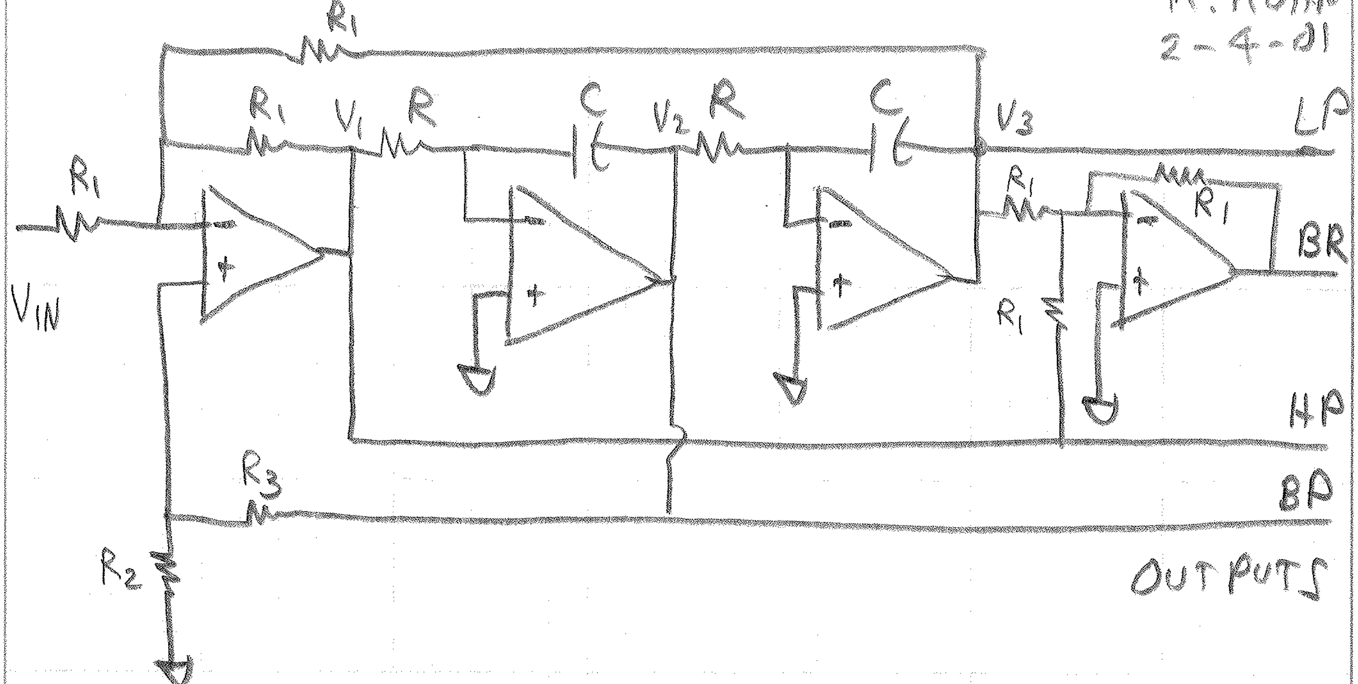
$$Q = \frac{1}{2}\sqrt{\frac{R_2}{R_1}}$$

1. LET $C_1 = C_2 = C \Longrightarrow$ SELECTED APPROPRIATE STANDARD VALUE
2. $R = \dfrac{1}{2\pi F C}$
3. $R_1 = R/2Q \Longrightarrow$ ROUNDED TO <u>NEAREST</u> STD VALUE
4. $R_2 = R * 2Q \Longrightarrow$ " " " " "

TRANSFER GAIN OF CIRCUIT AT RESONANT FREQUENCY IS $\underline{\underline{-2Q^2}}$

BIQUAD FILTER

K. KUHN
2-4-01



$$V_1 = -V_{IN} - V_3 + V_2\left(\frac{R_2}{R_2+R_3}\right)3 \qquad = V_{HP}$$

$$V_2 = \frac{-V_1}{RCs} \qquad = V_{BP}$$

$$V_3 = \frac{-V_2}{RCs} = \frac{V_1}{R^2C^2s^2} \qquad = V_{LP}$$

$$V_{BR} = -V_1 - V_3$$

SOLVE FOR V_{HP} FIRST

$$V_{HP} = -V_{IN} - \frac{V_{HP}}{R^2C^2s^2} - \frac{V_{HP}}{RCs}\left(\frac{R_2}{R_2+R_3}\right)(3)$$

$$V_{HP}R^2C^2s^2 = -V_{IN} - V_{HP} - V_{HP}\left(\frac{R_2}{R_2+R_3}\right)3RCs$$

$$V_{HP}\left(R^2C^2s^2 + 3\left(\frac{R_2}{R_2+R_3}\right)RCs + 1\right) = -V_{in}R^2C^2s^2$$

$$V_{HP} = \frac{-V_{IN} R^2 C^2 s^2}{R^2 C^2 s^2 + 3\left(\frac{R_2}{R_2+R_3}\right) RCs + 1}$$

$$\boxed{V_{HP} = \frac{-V_{IN} s^2}{s^2 + \frac{3}{RC}\left(\frac{R_2}{R_2+R_3}\right) s + \frac{1}{R^2C^2}}}$$

$$\boxed{\omega_n = \frac{1}{RC}}$$

$$2\zeta\omega_n = \frac{3}{RC}\left(\frac{R_2}{R_2+R_3}\right)$$

$$\zeta = \frac{\frac{3}{RC}\left(\frac{R_2}{R_2+R_3}\right)}{2\omega_n} = \frac{\frac{3}{RC}\left(\frac{R_2}{R_2+R_3}\right)}{2/RC} = \frac{3}{2}\left(\frac{R_2}{R_2+R_3}\right)$$

$$\zeta = \frac{3}{2}\left(\frac{1}{1+R_3/R_2}\right)$$

$$\zeta \Rightarrow 0 \text{ AS } R_3/R_2 \Rightarrow \infty$$

$$\zeta \Rightarrow 1.5 \text{ AS } R_3/R_2 \Rightarrow 0$$

$$Q = \frac{1}{2\zeta} = \frac{1}{2\left(\frac{3}{2}\right)\left(\frac{1}{1+R_3/R_2}\right)}$$

$$\boxed{Q = \frac{1+\frac{R_3}{R_2}}{3}}$$

$$3Q = 1 + R_3/R_2$$

$$\boxed{\frac{R_3}{R_2} = 3Q - 1}$$

The denominator in terms of Q

$$s^2 + \frac{3}{RC}\left(\frac{1}{1+\frac{R_3}{R_2}}\right)s + \frac{1}{R^2C^2}$$

$$= s^2 + \frac{3}{RC}\left(\frac{1}{1+3Q-1}\right)s + \frac{1}{R^2C^2}$$

$$= s^2 + \frac{3}{RC}\left(\frac{1}{3Q}\right)s + \frac{1}{R^2C^2}$$

$$= s^2 + \frac{s}{RCQ} + \frac{1}{R^2C^2}$$

$$\boxed{V_{HP} = \frac{-V_{IN}\,s^2}{s^2 + \frac{s}{RCQ} + \frac{1}{R^2C^2}}}$$

$$V_{BP} = V_{HP}\left(\frac{-1}{RCs}\right)$$

$$\boxed{V_{BP} = \frac{V_{IN}\,s/RC}{s^2 + \frac{s}{RCQ} + \frac{1}{R^2C^2}}}$$

$$V_{LP} = V_{BP}\left(\frac{-1}{RCs}\right)$$

$$\boxed{V_{LP} = \frac{-V_{IN}/R^2C^2}{s^2 + \frac{s}{RCQ} + \frac{1}{R^2C^2}}}$$

$$V_{BR} = -V_{HP} - V_{LP} = \frac{-\left(-V_{IN}s^2\right) - \left(-V_{IN}/R^2C^2\right)}{s^2 + \frac{s}{RCQ} + \frac{1}{R^2C^2}}$$

$$\boxed{V_{BR} = \frac{V_{IN}\left(s^2 + 1/R^2C^2\right)}{s^2 + \frac{s}{RCQ} + \frac{1}{R^2C^2}}}$$

MAGNITUDE OF TRANSFER FUNCTION AT $S = 1/RC$:

$$\left|\frac{V_{HP}}{V_{IN}}\right|_{s=1/RC} = Q$$

$$\left|\frac{V_{BP}}{V_{IN}}\right|_{s=1/RC} = Q$$

$$\left|\frac{V_{LP}}{V_{IN}}\right|_{s=1/RC} = Q$$

$$\left|\frac{V_{BR}}{V_{IN}}\right|_{s=1/RC} = 0$$

Q	R_3/R_2
1	2
2	5
5	14
10	29
20	59
50	149
100	299