Using Newton's Method to Solve the Classic Diode Problem

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Overview

This paper discusses a method to determine the current through silicon diodes when the diode is in series with a resistor and a known voltage is across the combination. At first, this problem looks trivial to solve but it turns out to be impossible to solve with any known mathematics. Thus, a numerical solution is required. With proper application of Newton's method, the solution will converge very accurately in only three to six iterations.

The Problem

Figure 1 shows the circuit. Figure 2 shows plots of the equations. The goal is to solve for the intersection of the diode function and the linear function for that is the operating point of the circuit. The lower curve on the plot is the difference between the diode function and the linear function. It should be obvious but notice that the curve crosses zero at exactly the intersection of the two functions. In the application of Newton's method we will be solving for the point where the lower curve crosses zero.

The equation for the intersection of the diode and linear function is:

$$0 = I_S * (e\hat{}(V_D/\eta V_T) - 1) - (V_A - V_D) / R \qquad \text{Eq. 1}$$

where:
I_S = reverse saturation current of diode
V_D = Voltage across diode
ηV_T = Thermal voltage, $nV_T = nkT/q$ where
- η = material constant (η is roughly between 1 and 2 for silicon)
- k = Boltzmann's constant = $1.38 * 10^{-23}$ Joules/°Kelvin
- T = temperature in °Kelvin
- q = magnitude of electron charge = $1.602 * 10^{-19}$ Coulombs

V_A = applied voltage
R = resistance in Ohms

There is no known way to arrange Equation 1 for a direct solution to V_D. Therefore, numerical methods will have to be used to iterate until V_D is determined to the desired precision.

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Newton's General Method for Solving Equations

Given an approximate solution, x_1, for the solution of $f(x) = 0$, then a more accurate solution, x_2, can be found by

$$x_2 = x_1 - f(x_1) / f'(x_1) \qquad \text{Eq. 2}$$

x_1 is an approximate solution which is near the actual solution (may be an estimate)
$f(x_1)$ is the value of the function of x evaluated at x1
$f'(x_1)$ is the value of the derivative of the function (with respect to x) evaluated at x1
x_2 is a more accurate solution

Each result of Equation 2 is fed back into Equation 2 for more refinement until the desired accuracy is achieved. If Equation 2 converges, then convergence is usually extremely fast. However, it is also possible for Equation 2 to diverge very fast. Divergence is usually caused by the initial guess being too far removed from the solution or if the derivative of the function becomes zero or changes sign near the solution.

Applying Newton's Method to Solve the Diode Problem

The derivative of Equation 1 is

$$(I_S/\eta V_T) * \exp(V_D/\eta V_T) + 1/R \qquad \text{Eq. 3}$$

Substituting Equations 1 and 3 into Equation 2 gives us Newton's method for this problem.

$$V_{D}\_new = V_D - \frac{I_S * (\exp(V_D/\eta V_T) - 1) - V_A/R + V_D/R}{(I_S/\eta V_T) * \exp(V_D/\eta V_T) + 1/R} \qquad \text{Eq. 4}$$

A starting estimate for V_D needs to be determined. There are two approaches to this. We note that V_D must be less than a value that would produce a diode current higher than V_A/R. Also, it is not possible for V_D to be greater than V_A. Therefore, the smaller of these two possible values will be used.

$$V_D = \eta V_T * \ln(1 + (V_A/R)/I_S) \qquad \text{Eq. 5}$$

if $V_D > V_A$ then $V_D = V_A$

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Now that the derivation is complete, we can simplify the process. Looking at Equation 4, the following factors can be computed once prior to starting the algorithm. There is no point in re-computing these constants with each iteration.

ηV_T, V_A/R, $I_S/\eta V_T$, $1/R$

Also, with each iteration, the expression, $\exp(V_D/\eta V_T)$, could be computed once for the numerator and then saved for reuse in evaluating the denominator.

The last issue to address is when to stop the algorithm. Testing indicates that the algorithm generally converges to six significant figures or better with only three to six iterations. Since the parameters, I_S, η, and T are generally not known to better than two or three significant figures, it makes little sense to attempt to compute V_D to greater precision. It may be simpler to just run the algorithm a set number of iterations rather than testing accuracy after each iteration. The following test can be used if desired to limit the number of iterations such that V_D_new is within 0.2% of the previous value.

Iterate until (absolute value $(V_D - V_D\_new) / (V_D + V_D\_new) < 0.001$)

An example problem

A certain small signal diode operating at 25°C with $I_S = 3.3$ nA and $\eta = 1.8$ is connected in the circuit shown in Figure 1 with an applied voltage of 0.8 Volts and a series resistor of 4700 Ohms. Find the current in the circuit. First, looking at Figure 2 we see that V_D is a little higher than 0.46 Volts and that the current is a little higher than 70 uA.

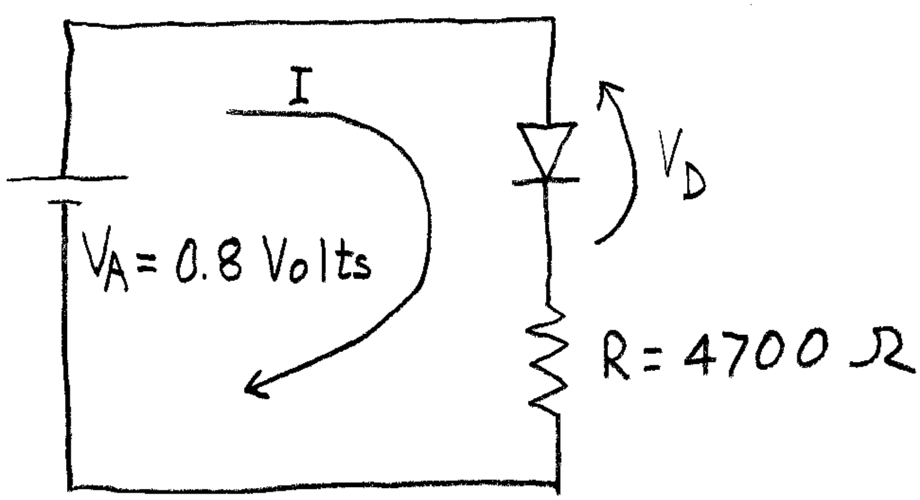

Figure 1: Diode circuit

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Step 1: Compute constant factors

V_T = $1.38*10^{-23} * (273 + 25) / 1.602*10^{-19}$ = <u>0.02567 Volts</u>
ηV_T = 1.8 * 0.02567 = <u>0.04621 Volts</u>
V_A/R = 0.8 / 4700 = <u>170 uA</u>
$I_S/\eta V_T$ = 3.3 nA / 0.04621 = <u>$7.141*10^{-8}$</u>
1/R = 0.000213

Step 2: Substitute into Equation 4 to make the iteration equation

$$V_{D}\_new = V_D - \frac{3.3\text{ nA} * (\exp(V_D/0.04621) - 1) - 170\text{ uA} + V_D/4700}{7.141*10^{-8} * \exp(V_D/0.04621) + 0.000213} \qquad \text{Eq. 6}$$

Step 3: Determine starting voltage

V_D = 0.04621 * ln(1 + 170 uA / 3.3 nA) = <u>0.501319</u>
We will use this value since it is less than 0.8

Step 4: Iterate Equation 6 until solution is found

V_D	Comments
0.501319	Initial estimate for V_D
0.474007	Improved estimate after first iteration
0.463003	Improved estimate after second iteration
0.461645	Improved estimate after third iteration (note the good accuracy here)
0.461627	Improved estimate after fourth iteration
0.461627	No more improvement (to six places) after fifth iteration

Using the final value of V_D the current is found to be <u>72 uA</u> – the solution to the problem. Note that these results confirm our initial observation from the plot.

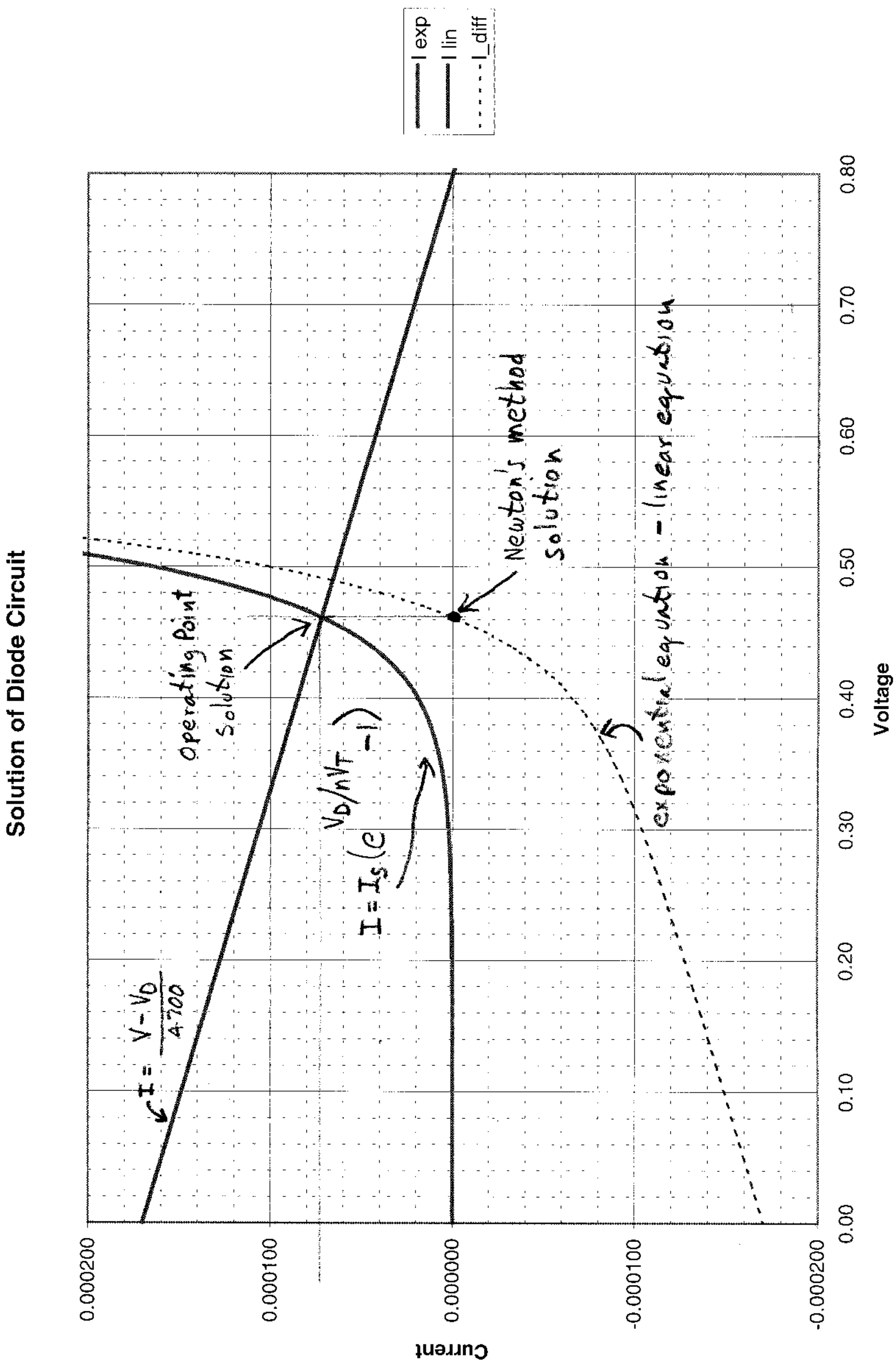

Figure 2: Graphical solution