NOISE IN RESISTORS

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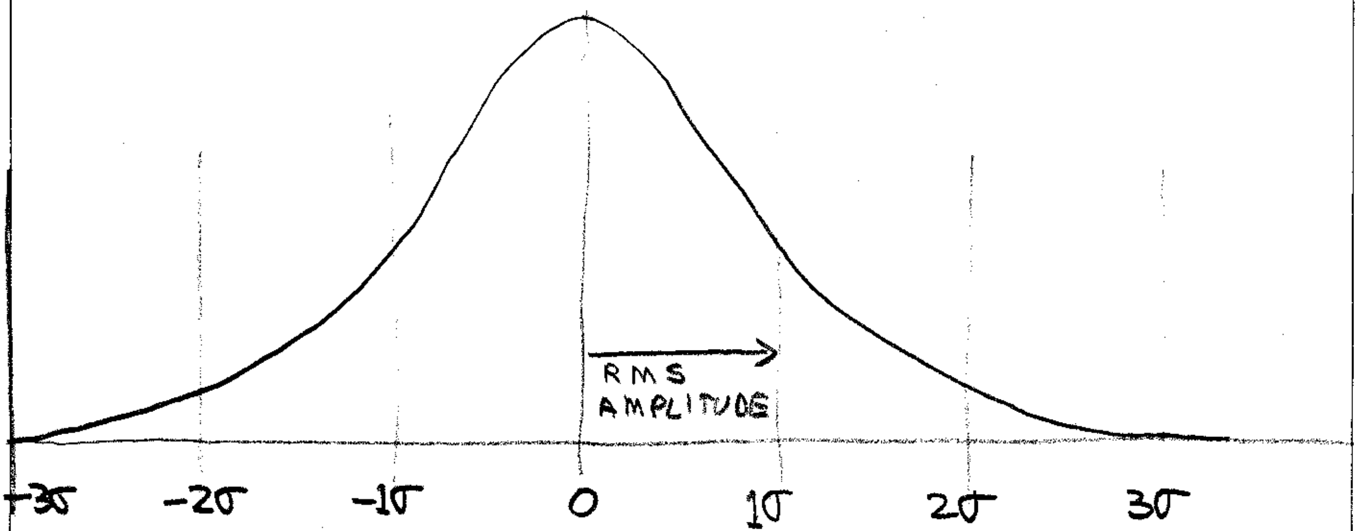
2-28-00

- AT ANY TEMPERATURE ABOVE 0°K THERE IS RANDOM MOTION OF CHARGES DUE TO THERMAL AGITATION. THE MOTION IS PROPORTIONAL TO TEMPERATURE. THE RESULT OF THIS MOTION IS A RANDOM VOLTAGE ACROSS THE RESISTOR, THIS RANDOM VOLTAGE IS CALLED NOISE. THE NOISE HAS A GAUSSIAN DISTRIBUTION OF AMPLITUDES.



THE AMPLITUDE DISTRIBUTION IS WITHIN ±3σ (99.7%) OF THE TIME. THIS MEANS THAT THE RMS VALUE OF THE NOISE IS APPROXIMATELY 1/6 THE PEAK TO PEAK VALUE. THE RMS VALUE OF THE NOISE CAN BE COMPUTED FROM;

$$V_n = \sqrt{4KT\Delta F R} \qquad \text{Eq 1}$$

- R — OHMS
- ΔF — HERTZ - EFFECTIVE BANDWIDTH IN HZ
- T — °K
- K — 1.38×10^{-23} J/°K BOLTZMANN'S CONSTANT

NOTE THAT $(KT\Delta F)$ HAS UNITS OF WATTS. FROM A COMBINATION OF OHM'S AND WATT'S LAW, $P = E^2/R$ OR $E = \sqrt{PR}$. NOTE THAT THE LATTER EQUATION IS JUST WHAT WE HAVE.

A FEW EXAMPLES; ($T = 300°K$ OR $27°C$).

① 600Ω, 20kHz BW (ΔF) (Typical audio system)

$V_n = 446 nV_{rms}$ $P_n = 331 aW$ OR $-125 dBm$

② $1 M\Omega$, 100 MHz (Typical oscilloscope)

$V_n = 1.3 mV_{rms}$ $P_n = 1.7 pW$ OR $-88 dBm$

↕ independent of resistance

③ 50Ω, 100 MHz

$V_n = 9.1 \mu V_{rms}$ $P_n = 1.7 pW$ OR $-88 dBm$

④ 50Ω, 3 kHz good shortwave receiver (voice)

$V_n = 50 nV_{rms}$ $P_n = 50 aW$ OR $-133 dBm$

NOISE REPRESENTS A LIMIT TO HOW WEAK A SIGNAL THAT CAN BE DETECTED. FOR RELIABLE PROCESSING A SIGNAL GENERALLY NEEDS TO BE STRONGER THAN THE NOISE BY TYPICALLY A FACTOR OF 2 BUT FOR AUDIO OR VIDEO SYSTEMS A FACTOR OF ABOUT 100 IS THE MINIMUM ACCEPTABLE (THE FACTORS ARE POWER RATIOS). THIS IS CALLED THE SIGNAL TO NOISE RATIO.

WHEN RESISTANCE IS NOT A VARIABLE AND BANDWIDTH IS, WE OFTEN MODIFY EQ 1 TO HAVE UNITS OF $V/\sqrt{Hz}$ (PRONOUNCED VOLTS PER ROOT HERTZ). WE USE A ΔF OF 1 Hz AND OBTAIN THE FOLLOWING.

$$V_n/\sqrt{Hz} = \sqrt{4KTR} \qquad \text{Eq 2.}$$

TO OBTAIN THE ACTUAL NOISE VOLTAGE, WE MULTIPLY BY THE SQUARE-ROOT OF THE BANDWIDTH BEING USED. CONSIDER EX. 3 & 4 OF BEFORE.

FOR A 50 Ω SYSTEM THE NOISE VOLTAGE IS 910 $pV/\sqrt{Hz}$, THEN FOR 100 MHz BW WE MULTIPLY BY $\sqrt{100\,MHz}$ TO OBTAIN 9.1 μV. FOR 3 KHz BW WE MULTIPLY BY $\sqrt{3\,KHz}$ TO OBTAIN 50 nV.

ONE WAY TO OBTAIN LOW NOISE IS TO COOL THE SYSTEM TO LOWER THE T FACTOR. LIQUID NITROGEN BOILS AT −196° C OR 77° K, BY COOLING DEVICES TO THIS TEMPERATURE, THE NOISE POWER IS REDUCED BY A FACTOR OF 300/77 = 3.9.

NOISE PROBLEMS (with answers)

($T = 300°K$)

1. WHAT IS THE HIGHEST BANDWIDTH THAT CAN BE USED TO KEEP THE NOISE POWER BELOW −140 dBm? (−140 dBm = 10 aW → 2.4 kHz)

2. FIND V_n FOR BW = 1 MHz AND R = 1 MΩ. ALSO FIND P_n IN WATTS AND dBm. (129 nV, 4.1 fW, −114 dBm)

3. WHAT IS THE HIGHEST BANDWIDTH THAT CAN BE USED IN A 50 Ω SYSTEM SO THAT THE NOISE VOLTAGE IS NO MORE THAN 10 μVrms? (121 MHz)

4. IF A SYSTEM HAS A NOISE VOLTAGE OF 1 μV/$\sqrt{Hz}$, FIND THE NOISE VOLTAGE FOR A BANDWIDTH OF 20 kHz AND ALSO FOR A BANDWIDTH OF 100 MHz. (Ans 141 μV, 10 mVrms)

5. IF THE MEASURED NOISE (ON AN OSCILLOSCOPE) IS 100 mVpp AND THE BANDWIDTH IS 50 kHz, WHAT IS THE NOISE VOLTAGE IN RMS AND IN RMS VOLTS PER ROOT HERTZ? (16.7 mV, 74.7 μV/$\sqrt{Hz}$)

6. IF THE BANDWIDTH IS 1 Hz, WHAT IS THE NOISE LEVEL IN dBm? (Ans −174 dBm)

Resistor Noise

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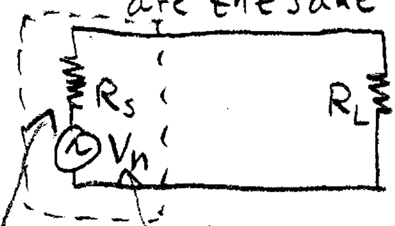
Both resistors are the same value



← Theoretical noiseless resistor } receives P_n below

thermal noise voltage

source resistance with thermal noise

> our goal is to determine P_n or V_n

Noise voltage delivered to $R_L = V_n \dfrac{R_L}{R_S + R_L} = \dfrac{V_n}{2}$

Noise power delivered to $R_L = \dfrac{\left(\frac{V_n}{2}\right)^2}{R_L}$ ← $= R$

Note that:

$P = \dfrac{V^2}{R}$ and $V = \sqrt{PR}$

Thus:

$\dfrac{V_n^2}{4} = P_n R$ and $\boxed{V_n = \sqrt{4 P_n R}}$

$\boxed{P_n = KTB}$

$K = 1.38 \times 10^{-23}$ J/°K (Boltzmann's constant)
$T =$ temperature in °K
$B =$ Noise equivalent bandwidth in Hz

Thus: KTB has units of J/s or Watts

$\boxed{V_n = \sqrt{4KTBR}}$

↑ open circuit voltage

(↑ s: $Hz = \dfrac{1}{s}$)