Op-Amp Circuit Analysis

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Introduction

This article demonstrates the fundamental concepts of analysis to determine the transfer function of an op-amp circuit. First, the classical analysis method which includes the finite op-amp gain term, Av is shown. Then the easier ideal analysis method which assumes that Av is infinity is shown.

The student should be aware of the classical method because when pushing circuit performance to the limit the ideal case will have too much error. However, the majority of circuits are not pushed to the limit and the error of ideal analysis is negligibly small. Ideal analysis is preferred most of the time because it is easier, faster, and less prone to mistakes.

The Classical Analysis Method

There are four fundamental steps to the classical analysis method as illustrated below on the two most common op-amp circuits. The steps are always the same for any circuit.

Simple Non-inverting Amplifier

The non-inverting amplifier is the easiest to analyze and is shown in Figure 1 below.

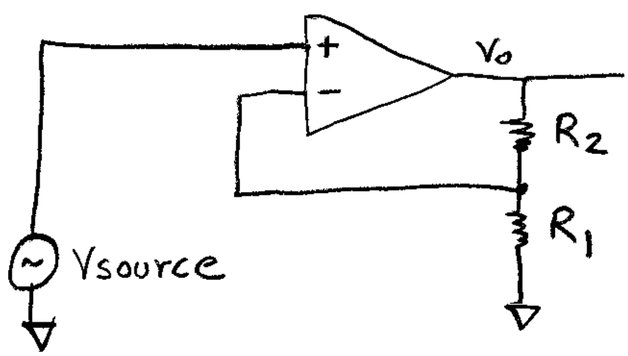

Figure 1: Simple Non-Inverting Amplifier

Step 1: Write equations describing the two inputs

Vin+ = Vsource — This one is usually obvious

$$Vin- = \frac{Vo * R1}{R1 + R2}$$

Simple voltage divider on output

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Step 2:Substitute the equations into the op-amp model

$$V_o = (V_{in+} - V_{in-}) * A_v$$ Simple op-amp model

$$V_o = \left(V_{source} - \frac{V_o * R1}{R1 + R2} \right) * A_v$$ With substitution

Step 3:Solve the resulting equation for Vo in terms of the input(s)

First we gather the Vo terms on the left side.

$$V_o * \left(1 + \frac{A_v * R1}{R1 + R2}\right) = A_v * V_{in}$$

Then we solve for Vo.

$$V_o = \frac{A_v * V_{in}}{1 + \dfrac{A_v * R1}{R1 + R2}}$$ Correct result but messy

Step 4: Simplify the resulting solution for maximum clarity

Although correct, this first solution is hard to interpret since an unknown large value, Av, is in the numerator. Large unknown values should be in the denominator. We now divide both numerator and denominator by Av.

$$V_o = \frac{V_{in}}{\dfrac{1}{A_v} + \dfrac{R1}{R1 + R2}}$$

This version is better as it clearly shows that a large Av has minimal effect on the result. However, it is still a bit difficult to understand. Applying the concept of “the end justifies the means” we will multiply by the inverse of the resistor fraction.

$$Vo = \frac{Vin * \frac{(R1 + R2)}{R1}}{\frac{(R1 + R2)}{Av * R1} + 1}$$

It might look like we have made things more complicated but this move has positioned us for a significant simplification.

$$Vo = \frac{Vin * (1 + (R2/R1))}{\frac{1 + (R2/R1)}{Av} + 1}$$ The final result

This is our final equation. Note that the gain is simply one plus the ratio of the resistors. Note also that if Av is very large in comparison to this gain then the denominator term goes to 1. Both of these concepts were hard to visualize in the first solution.

Lets work a typical example using R1 = 1K, R2 = 100K, and Av = 50,000. This results in

$$Vo = \frac{Vsource * 101}{\frac{101}{50{,}000} + 1} = \frac{Vsource * 101}{1.0020} \sim= Vsource * 101$$

Simple Inverting Amplifier

The inverting amplifier is slightly more complicated to analyze and is shown in Figure 2 below.

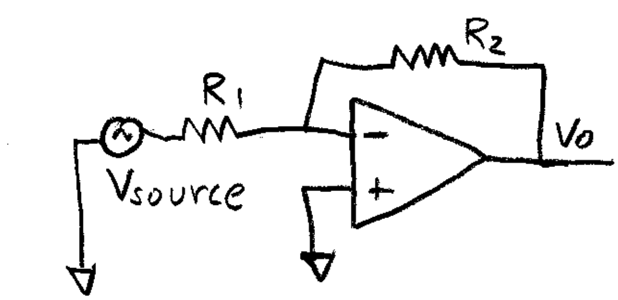

Figure 2: Simple Inverting Amplifier

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Step 1: Write equations describing the two inputs

Vin+ = 0

We apply superposition for the next step

$$Vin- = \frac{Vsource * R2}{R1 + R2} + \frac{Vo * R1}{R1 + R2}$$

Review superposition until you understand

Step 2: Substitute the equations into the op-amp model

Vo = (Vin+ - Vin-) * Av

$$Vo = \left(0 - \frac{Vsource * R2}{R1 + R2} - \frac{Vo * R1}{R1 + R2}\right) * Av$$

Step 3: Solve the resulting equation for Vo in terms of the input(s)

First we gather the Vo terms on the left side.

$$Vo * \left(1 + \frac{Av * R1}{R1 + R2}\right) = \frac{-Vsource * Av * R2}{R1 + R2}$$

Then we solve for Vo.

$$Vo = \frac{\dfrac{-Vsource * Av * R2}{R1 + R2}}{1 + \dfrac{Av * R1}{R1 + R2}}$$

Correct result but messy

Step 4: Simplify the resulting solution for maximum clarity

Although correct, this first solution is hard to interpret since an unknown large value, Av, is in the numerator. Large unknown values should be in the denominator. We now divide both numerator and denominator by Av and also multiply by R1 + R2 to clean things up.

$$V_o = \frac{-V_{source} * R2}{\dfrac{R1 + R2}{A_v} + R1}$$

This is better but the equation still lacks clarity. We now divide numerator and denominator by R1.

$$V_o = \frac{-V_{source} * (R2/R1)}{\dfrac{1 + (R2/R1)}{A_v} + 1} \qquad \text{Final result}$$

This is our final equation. Note that the gain is simply the ratio of the resistors and is negative because the output phase is inverted relative to the input. Note also that if Av is very large in comparison to this gain then the denominator term goes to 1. Both of these concepts were hard to visualize in the first solution.

Lets work a typical example using R1 = 1K, R2 = 100K, and Av = 50,000. This results in

$$V_o = \frac{-100 * V_{source}}{\dfrac{101}{50{,}000} + 1} = \frac{-100 * V_{source}}{1.0020} \approx -100 * V_{source}$$

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Ideal Analysis Method

The ideal analysis method is inspired by the fact that in the preceding we strove to create equations that indicated that Av can be ignored if it is sufficiently high. For ideal analysis, rather than do the cumbersome algebra to manipulate Av, we assume from the start that Av is high – ideally infinite. The method is best illustrated by example so we will repeat the first two examples below.

Simple Non-inverting amplifier

Step 1:Write equations describing the two inputs

Vin+ = Vsource

$$Vin- = \frac{Vo * R1}{R1 + R2}$$

Note that this step is identical to the classical method.

Step 2: Set Vin+ = Vin- and solve for Vo

Vin+ = Vin-

$$Vsource = \frac{Vo * R1}{R1 + R1}$$

Solving for Vo gives

$$Vo = \frac{Vsource * (R1 + R2)}{R1}$$

Step 3: Simply the resulting solution for maximum clarity

All we need to do for this equation is to convert the resistor fraction to ratio form.

Vo = Vsource * (1 + R2/R1)

Note how simple this solution was compared to the classical method.

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Simple Inverting Amplifier

Step 1: Write equations describing the two inputs

Vin+ = 0

We apply superposition for the next step

$$Vin- = \frac{Vsource * R2}{R1 + R2} + \frac{Vo * R1}{R1 + R2}$$

Step 2: Set Vin+ = Vin- and solve for Vo

Vin+ = Vin-

$$0 = \frac{Vsource * R2}{R1 + R2} + \frac{Vo * R1}{R1 + R2}$$

Solving for Vo gives

Vo = -Vsource * (R2/R1)

Step 3: Simply the resulting solution for maximum clarity

For this example there is nothing more to do as the answer is already in the desired ratio form.

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Applying the results

The ideal analysis method is very easy to perform. It can be shown that the vast majority of op-amp circuits are no more than combinations of non-inverting and inverting forms. Thus, if you recognize the form you already have the solution – an even further simplification. This is best illustrated by solving a four-resistor op-amp circuit as shown in Figure 3.

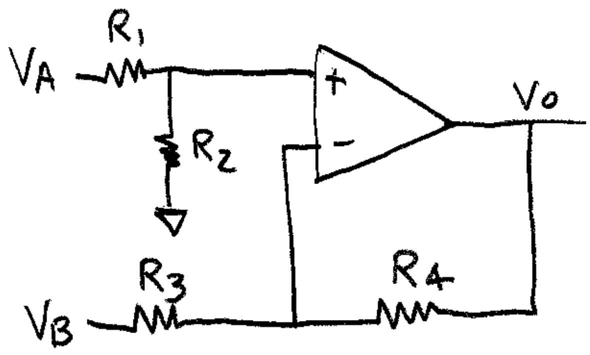

Figure 3: Two input, four resistor circuit

We should recognize that R1 and R2 form a voltage divider for VA and that R3 and R4 form a non-inverting amplifier for VA and an inverting amplifier for VB (we are using superposition). Thus we can write

$$Vo = VA * \frac{R2}{R1 + R2} * (1 + R4/R3) - VB * (R4/R3)$$

We did not even need steps 1 and 2. Since we like to have our results in ratio form then we apply step 3 to produce

$$Vo = VA * \frac{1 + (R4/R3)}{1 + (R1/R2)} - VB * (R4/R3)$$

This was a fairly complicated circuit and we solved it in only a few steps. The student should solve this circuit using the classical approach as an excellent example to perfect that skill.

While we are on this example, lets see what happens if we let R4 = R2 and R3 = R1. After a little simplification we have the result

Vo = (VA – VB) * (R2/R1)

We now have an amplifier that multiplies the difference between two voltages by a factor we set via a resistor ratio. Observe that this equation is very similar to the simple model for an op-amp except that the gain term is a small finite value we have direct control of. This gain term is often set to 1.0 to build a simple subtractor. Other popular factors are 2.0 and 10.0.

Solutions with complex impedances

It can be shown that the result is general and the resistors in the preceding examples can be replaced with complex impedances. Two classic examples are the integrator and differentiator. We will first analyze the integrator in Figure 4 by direct application.

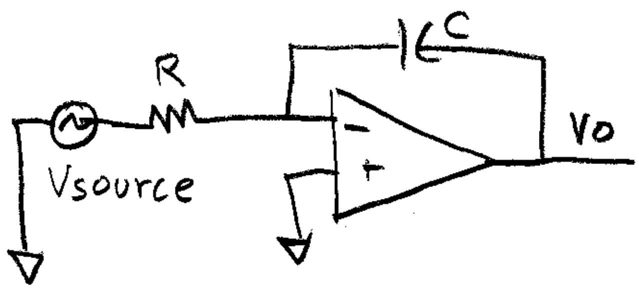

Figure 4: Integrator circuit

Vo = -Vsource * (Z2/Z1) = -Vsource * (1/Cs) / R = -Vsource / (RCs)

The differentiator in Figure 5 is analyzed the same way.

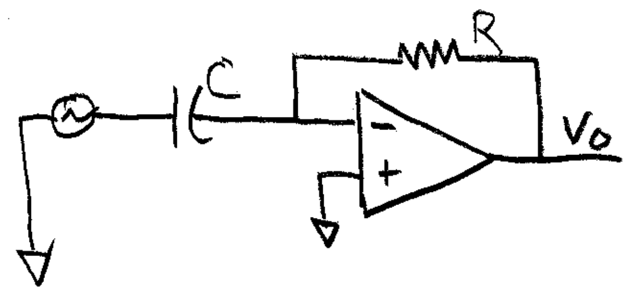

Figure 5: Differentiator circuit

Vo = -Vsource * (Z2/Z1) = - Vsource * R / (1/Cs) = -Vsource * RCs

Conclusions

It is easy to place the appropriate feedback network in an op-amp circuit to create a wide variety of mathematical operations. These transfer functions are easily derived using ideal analysis. The few examples shown here point to the reason for developing a universal amplifier building block known as an operational amplifier.