The s-plane and Standard 2nd order form

K. Kuhn
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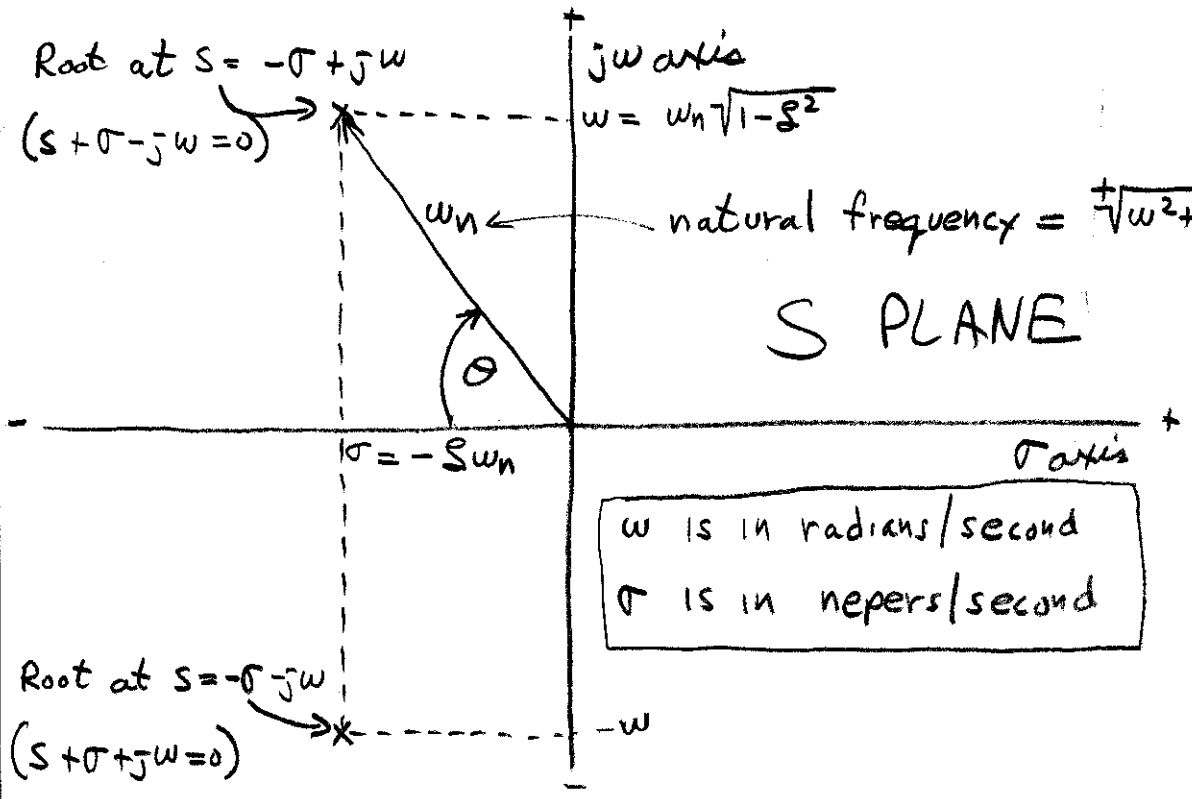

EQUATION IN s:

$$(s+\sigma-j\omega)(s+\sigma+j\omega) = s^2 + \sigma s + j\omega s + \sigma s + \sigma^2 + j\omega\sigma - j\omega s - j\omega\sigma + \omega^2$$

$$= s^2 + 2\sigma s + \omega^2 + \sigma^2$$

STANDARD FORM FOR 2nd ORDER EQUATION:

$$s^2 + 2\zeta\omega_n s + \omega_n^2$$

ζ = damping ratio
ω_n = natural frequency

THE ROOTS ARE:

$$s = \frac{-2\zeta\omega_n \pm \sqrt{(2\zeta\omega_n)^2 - 4\omega_n^2}}{2} = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$$

IN FACTOR FORM:

$$(s - \zeta\omega_n - j\omega_n\sqrt{1-\zeta^2})(s - \zeta\omega_n + j\omega_n\sqrt{1-\zeta^2})$$

sometimes Q is used instead of ζ

THEREFORE:

$$\sigma = -\zeta\omega_n \qquad \zeta = \frac{-\sigma}{\pm\sqrt{\omega^2+\sigma^2}} = \cos\theta = \frac{1}{2Q}$$

$$\omega = \pm\omega_n\sqrt{1-\zeta^2} \qquad \omega_n = \pm\sqrt{\omega^2+\sigma^2}$$